Reionization and the cosmic dawn

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- current problems

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- Asynchronous evolution

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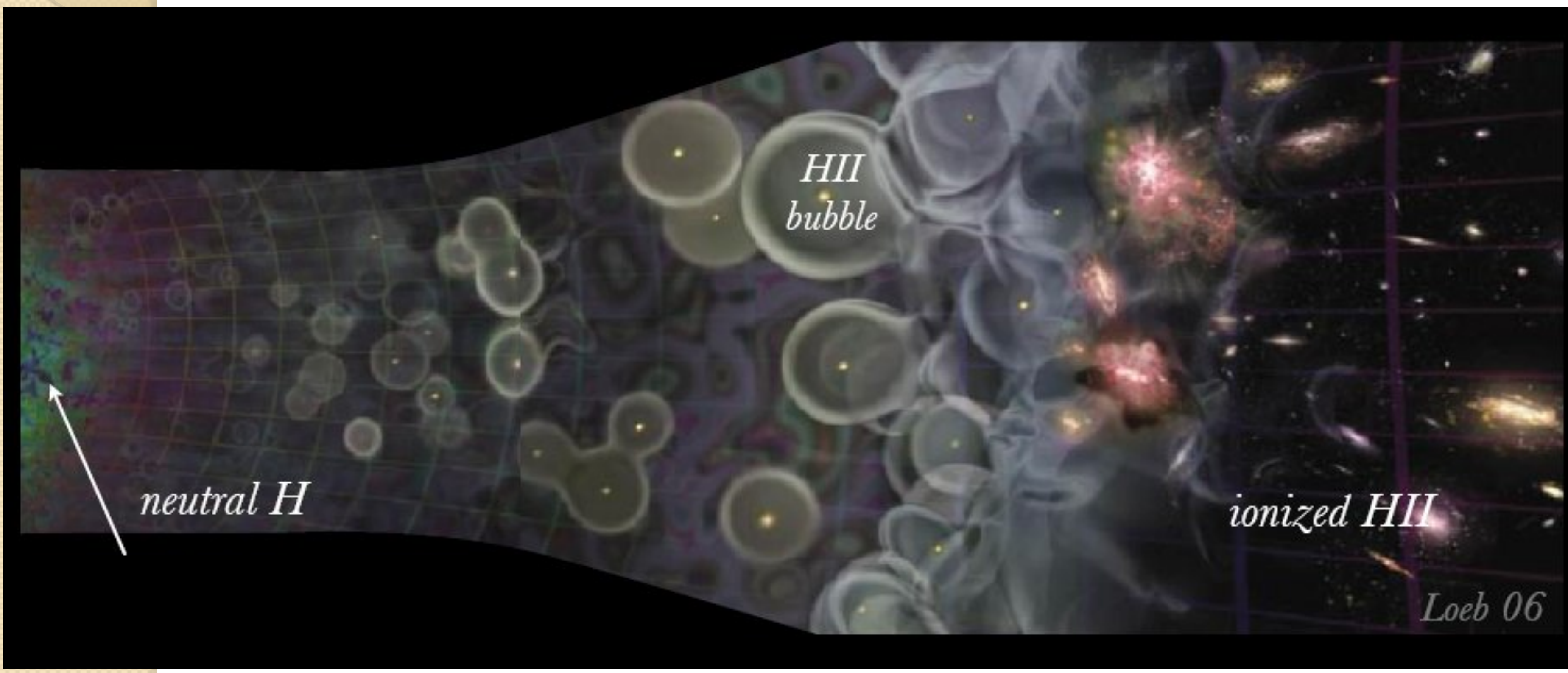
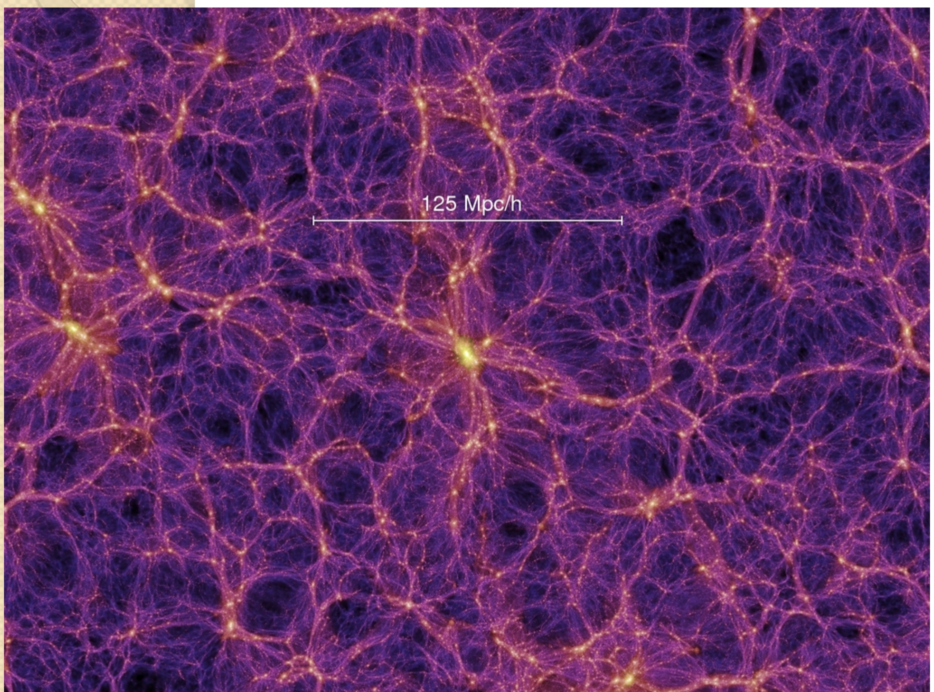
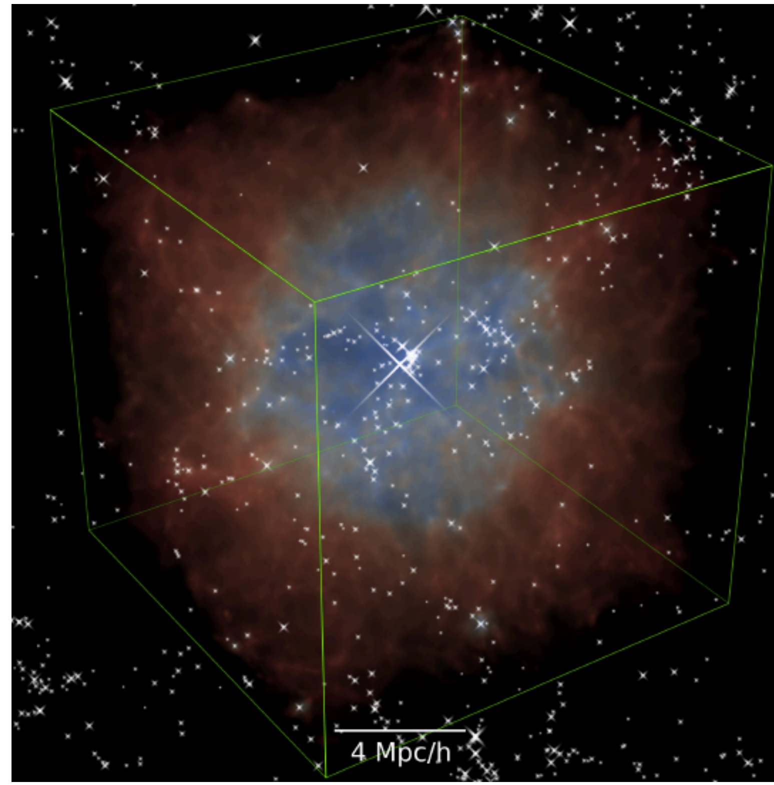
Epoch of reionization



Reionization simulation





N-body simulation

Radiative transfer simulation

Radiative Transfer equation

The complete RT equation

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t}+\frac{\hat{n}\cdot\nabla I_\nu}{\bar{a}}-\frac{H(t)}{c}\left(\nu\frac{dI_\nu}{d\nu}-3I_\nu\right)=j_\nu(\mathbf{x},\hat{n})-\alpha_\nu(\mathbf{x},\hat{n})I_\nu$$

In numerical simulation, we approximate it to…

$$\hat{n}\cdot\nabla I_\nu = j_\nu(\mathbf{x},\hat{n})-\alpha_\nu(\mathbf{x},\hat{n})I_\nu$$

But still it is an expensive calculation since it is a 7-dimensional problem

Radiative Transfer code - C^2-Ray

1st version (Mellema, 2006) – grid-based, photon conserving, hydrogen only, short characteristic,

On the spot approximation, iteration of the photo equations using averaged results

2nd version (Friedrich, 2012) – helium inclusion, multi-frequency approach, secondary ionization

3rd version (Lee, in preparation) – …….

C^2-Ray

We solve the ionization and thermal equations

$$\frac{dx}{dt} = (1-x)(\Gamma + n_e C_H) - x n_e \alpha_H \qquad \frac{\partial u}{\partial t} = \mathcal{H} - \mathcal{C}$$

The solutions are only accurate if the

1. Photoionization rate (Γ) and
2. Photoheating rate (H)

are estimated correctly

$$\Gamma_{HI}(\tau_\nu) = \int_{\nu_{HI}}^{\infty} \frac{L_\nu}{h\nu} \frac{e^{-\tau_\nu}(1-e^{-\tau_\nu})}{n_{HI}} \frac{1}{V_{shell}} \, d\nu$$

$$\mathcal{H}_{HI}(\tau_\nu) = \int_{\nu_{HI}}^{\infty} \frac{L_\nu}{h\nu} e^{-\tau_\nu}(1-e^{-\tau_\nu}) \frac{h(\nu-\nu_{HI})}{V_{shell}} \, d\nu$$

Problem of C²-Ray

- C2-Ray uses the below Γ and H

$$\Gamma_{HI}(\langle\tau_\nu\rangle) = \int_{\nu_{HI}}^{\infty} \frac{L_\nu}{h\nu} \frac{e^{-\langle\tau_\nu\rangle}(1-e^{-\langle\tau_\nu\rangle})}{\langle n_{HI}\rangle} \frac{1}{V_{shell}} \, d\nu$$

$$\mathcal{H}_{HI}(\langle\tau_\nu\rangle) = \int_{\nu_{HI}}^{\infty} \frac{L_\nu}{h\nu} e^{-\langle\tau_\nu\rangle}(1-e^{-\langle\tau_\nu\rangle}) \frac{h(\nu-\nu_{HI})}{V_{shell}} \, d\nu$$

The correct Γ and H are

$$\langle\Gamma_{HI}(\tau_\nu)\rangle = \int_{\nu_{HI}}^{\infty} \frac{L_\nu}{h\nu} \left\langle \frac{e^{-\tau_\nu}(1-e^{-\tau_\nu})}{n_{HI}} \right\rangle \frac{1}{V_{shell}} \, d\nu$$

$$\langle\mathcal{H}_{HI}(\tau_\nu)\rangle = \int_{\nu_{HI}}^{\infty} \frac{L_\nu}{h\nu} \langle e^{-\tau_\nu}(1-e^{-\tau_\nu})\rangle \frac{h(\nu-\nu_{HI})}{V_{shell}} \, d\nu$$

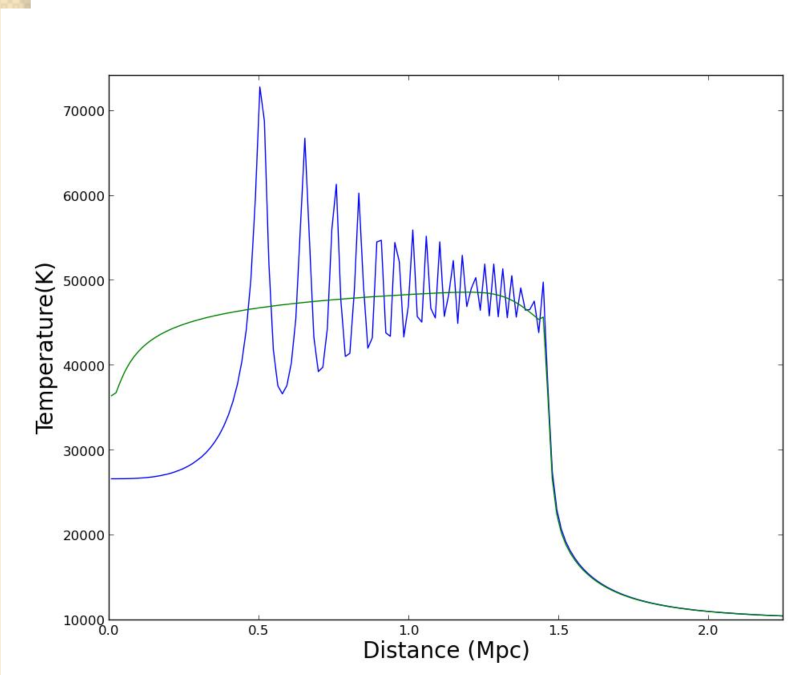
However, C2-Ray uses long time-step, it leads to

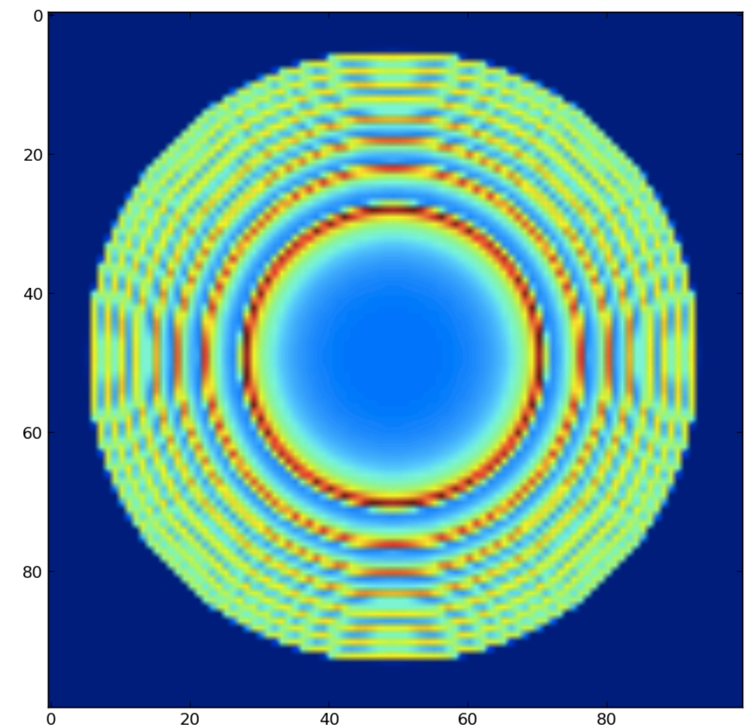
$$\langle\Gamma_{HI}(\tau_\nu)\rangle \approx \Gamma_{HI}(\langle\tau_\nu\rangle) \qquad \langle\mathcal{H}_{HI}(\tau_\nu)\rangle \gg \mathcal{H}_{HI}(\langle\tau_\nu\rangle)$$

Problem of C²-Ray

Ionization evolution : long time-step is ok.

Thermal evolution : long time-step is problematic.





We need to use a shorter time-step

Traditional time-step

Time-step depends on timescale of some physical process

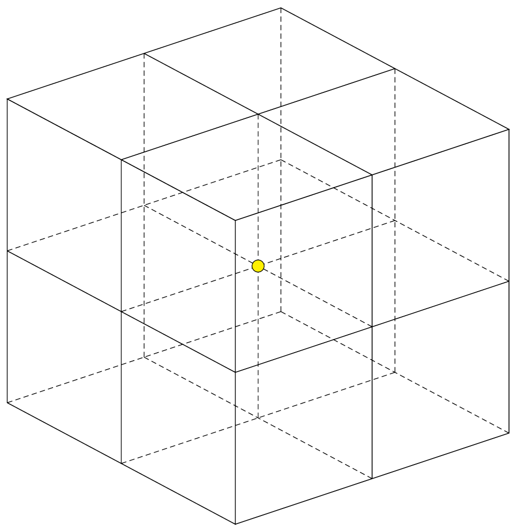
1. Ionization rate
2. Heating
3. Hydrodynamic

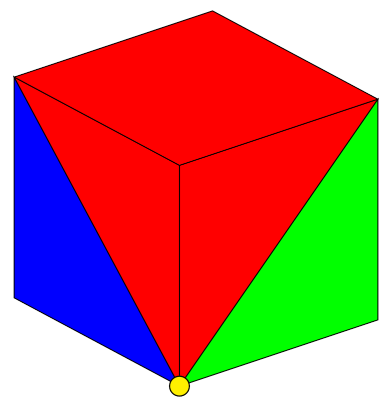
Scan through entire grid, choose the minimum one.

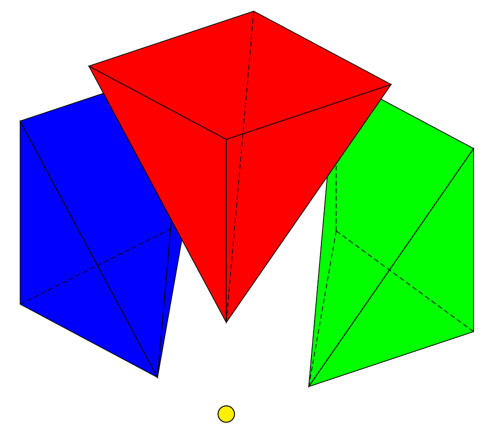
Adaptive time-step scheme

- Our adaptive time-step bases on one principle: Ionization front does not move more than one cell after N time-step
- N is a parameter determined upon the time of calculation
- It gives more control over the evolution
- We need to identify the ionization front cells that can be reached by each source
- Since we do this procedure in every time-step, we must contrive some quick method

Adaptive time-step scheme





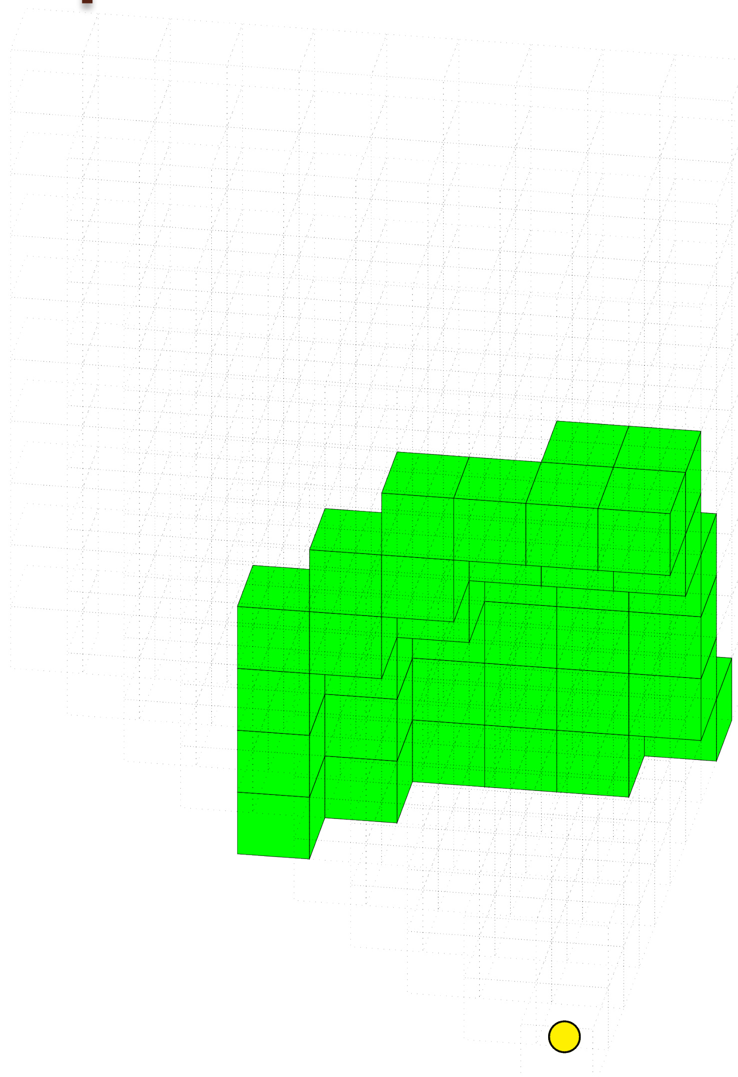


Domain decomposition

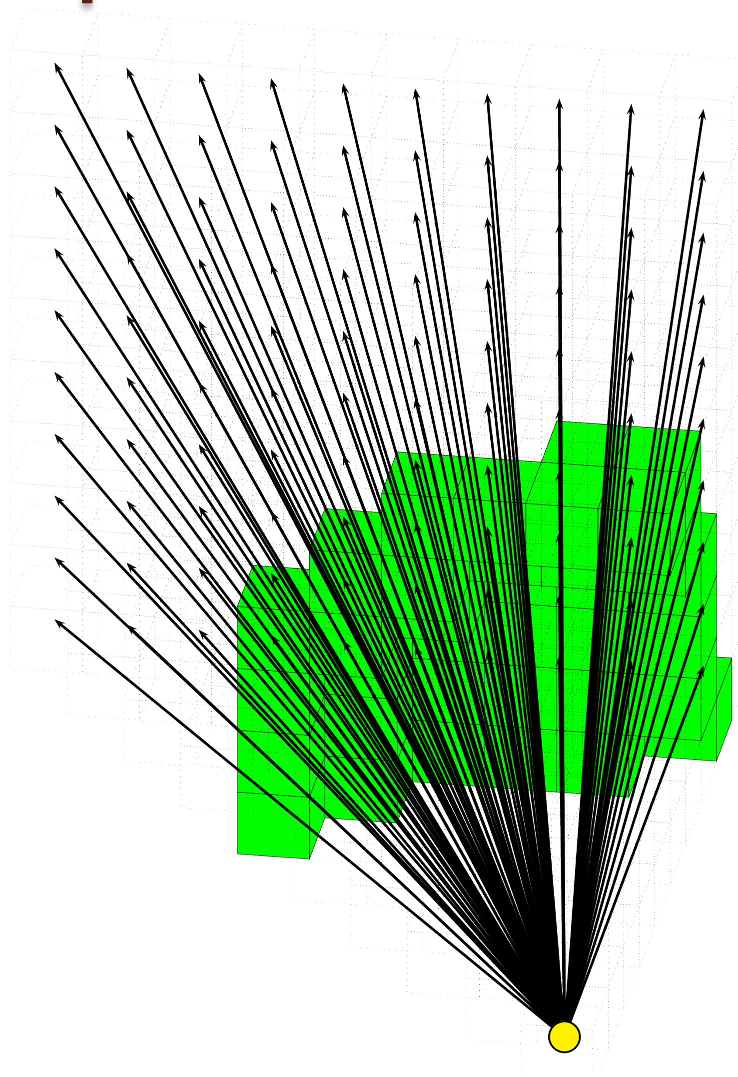
1 cube → 24 pyramids

Adaptive time-step scheme

Take one pyramid as an example, the green cells are ionization front, are they all reached by the yellow source?

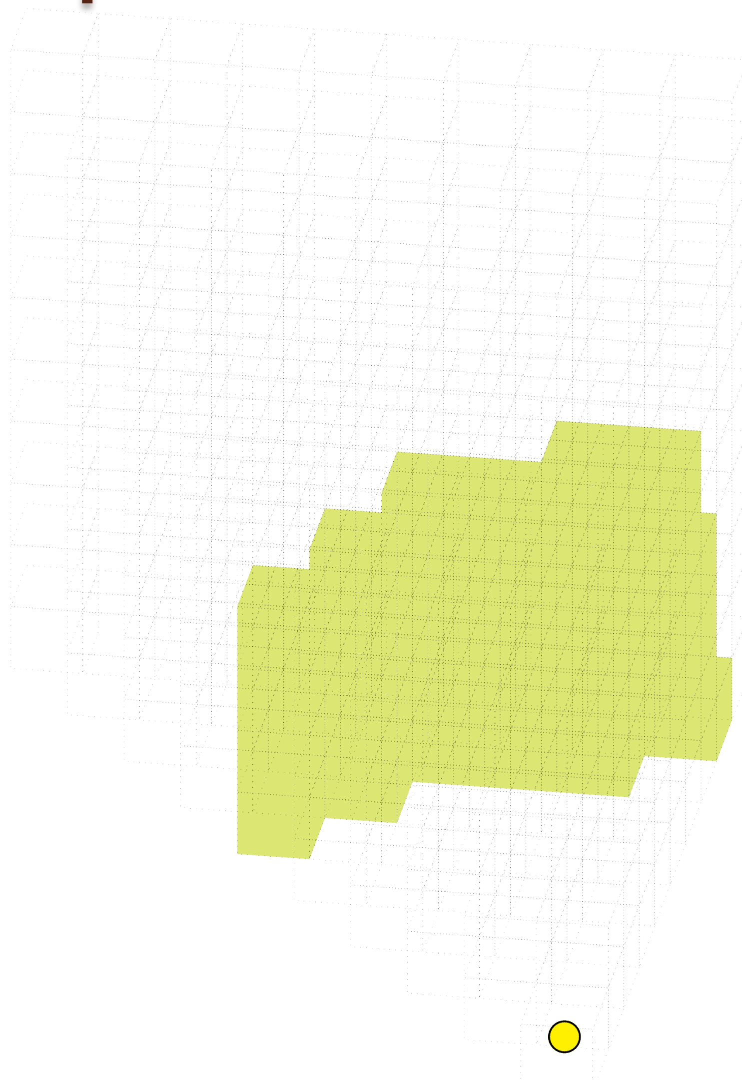


Adaptive time-step scheme



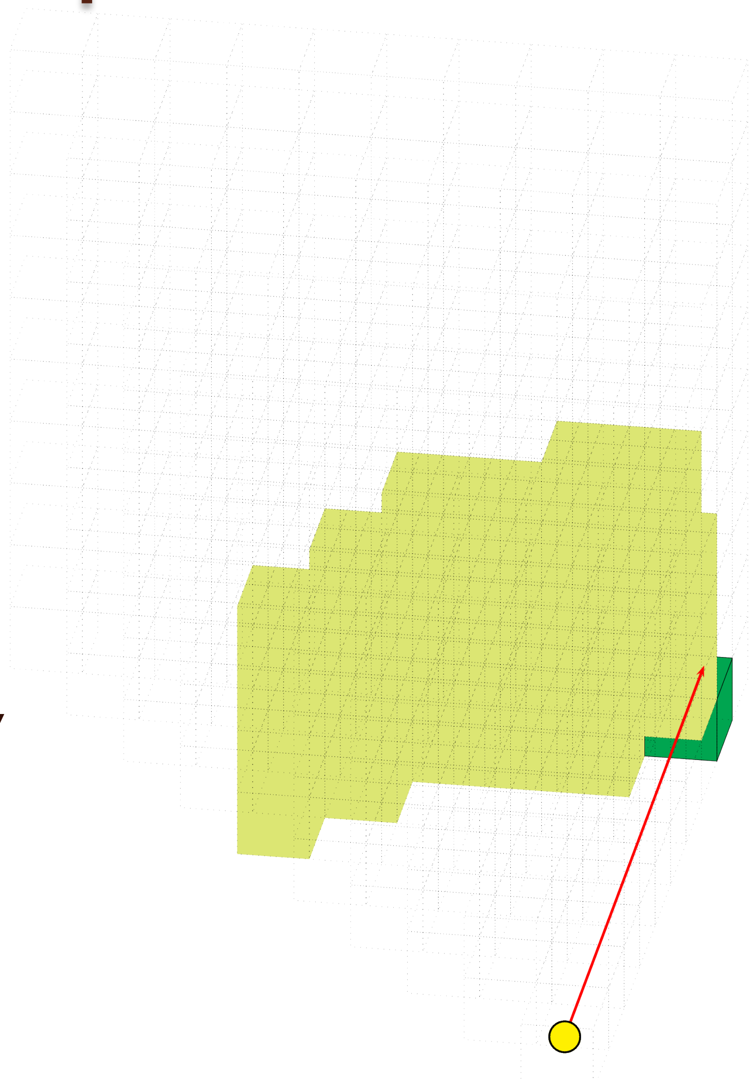
We can send many rays out and check the first green cells met by each ray, BUT….

Adaptive time-step scheme



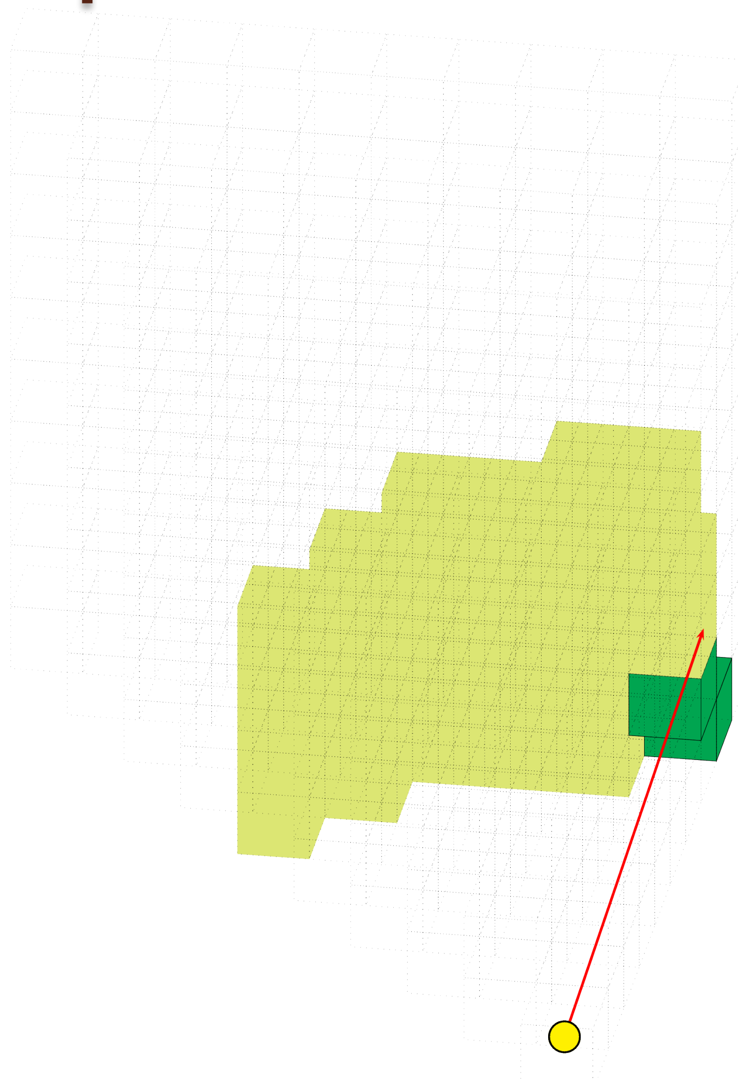
Our algorithm !!

Adaptive time-step scheme



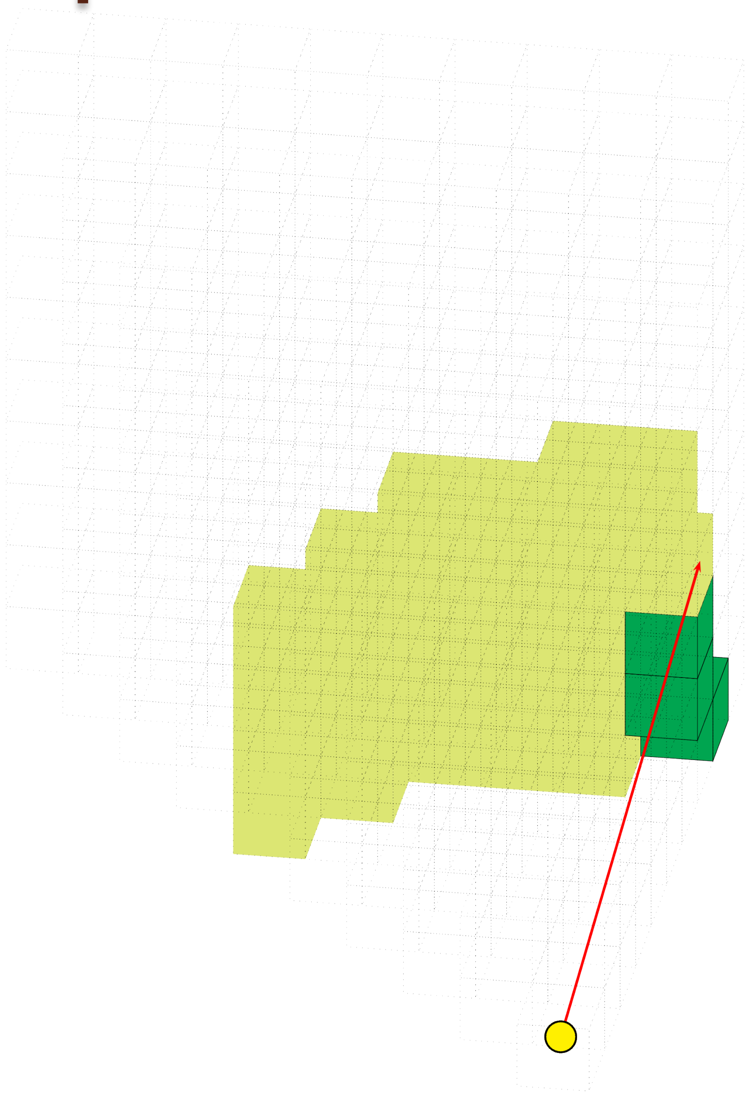
We send the first ray and identify the ionization front cell met by this ray

Adaptive time-step scheme



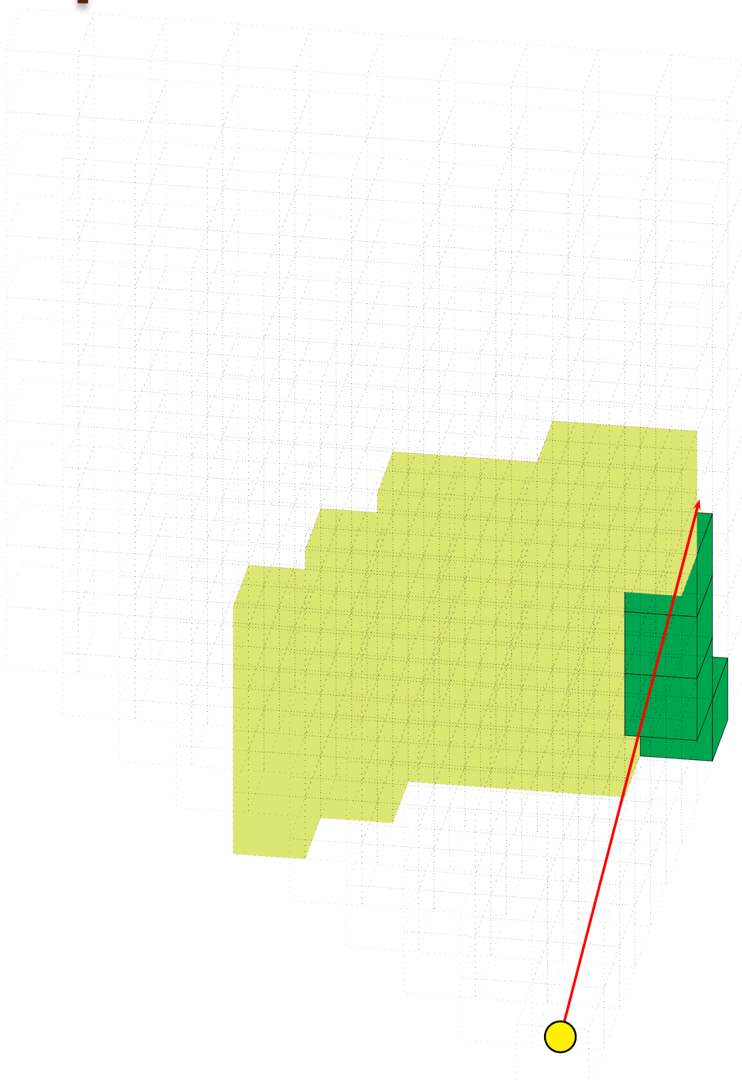
We send the second ray so that it passes just above the previous met cell

Adaptive time-step scheme



and so on

Adaptive time-step scheme



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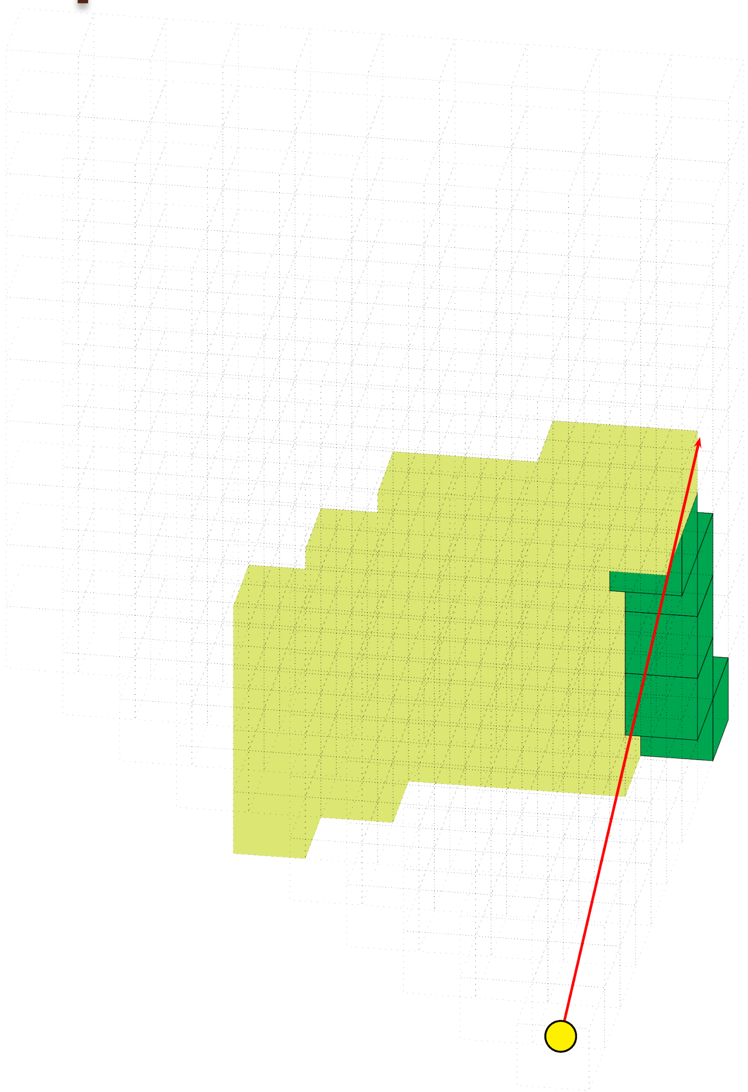
Adaptive time-step scheme



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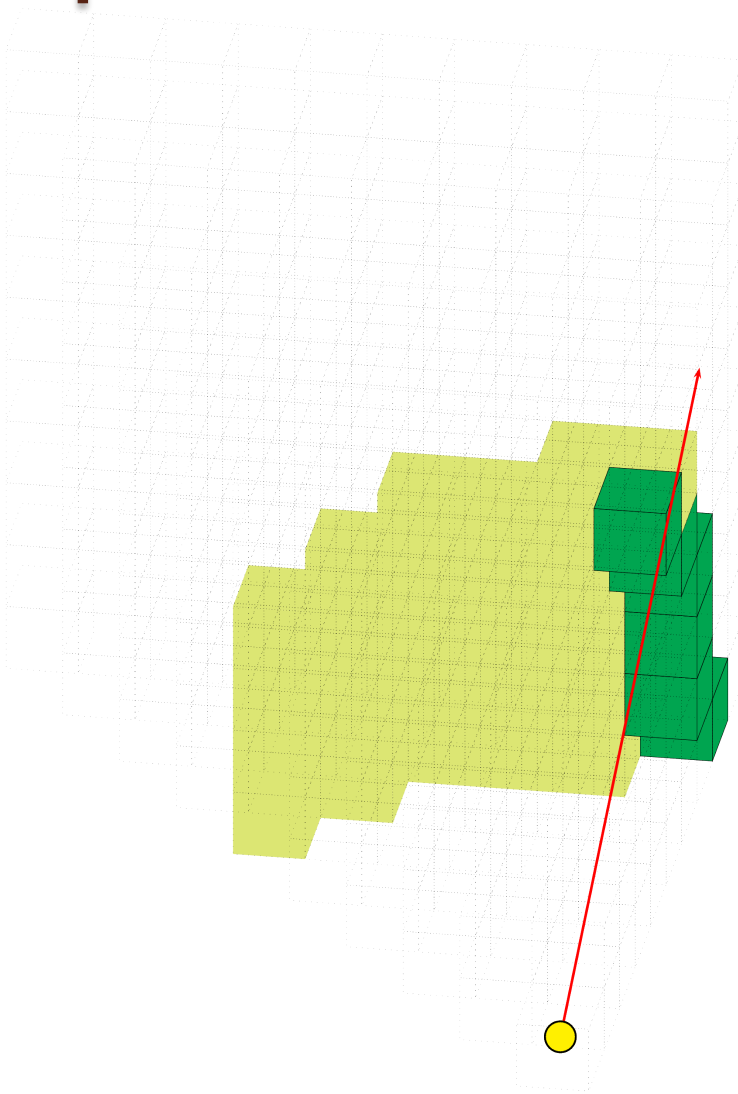
Adaptive time-step scheme



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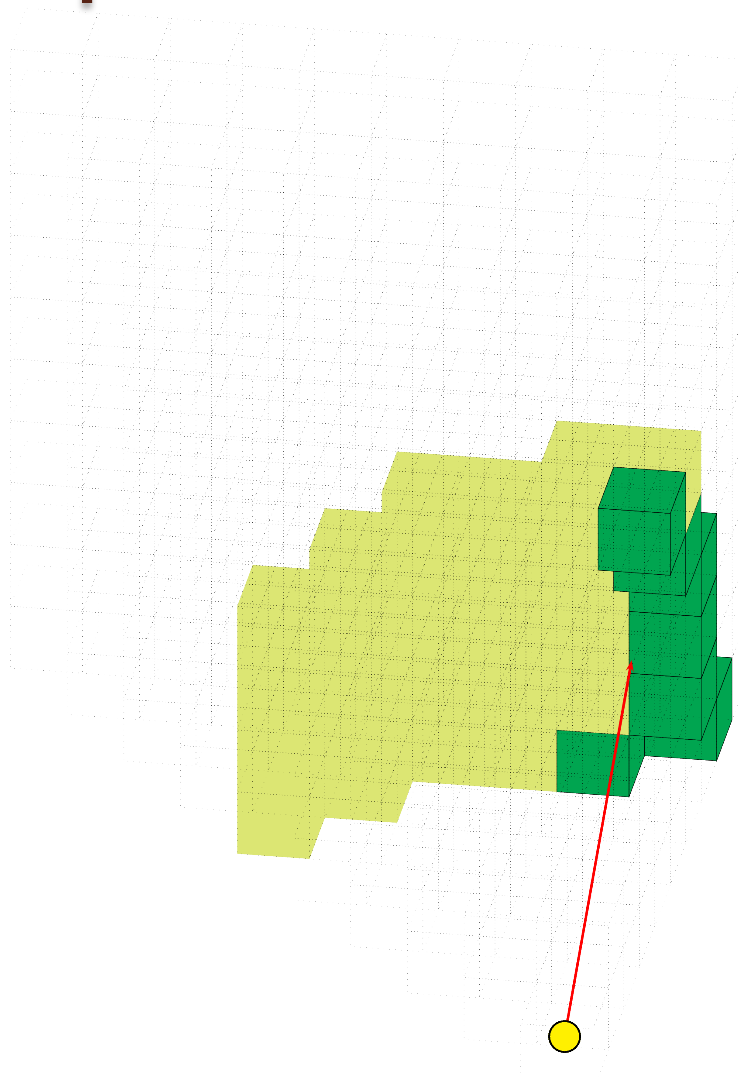
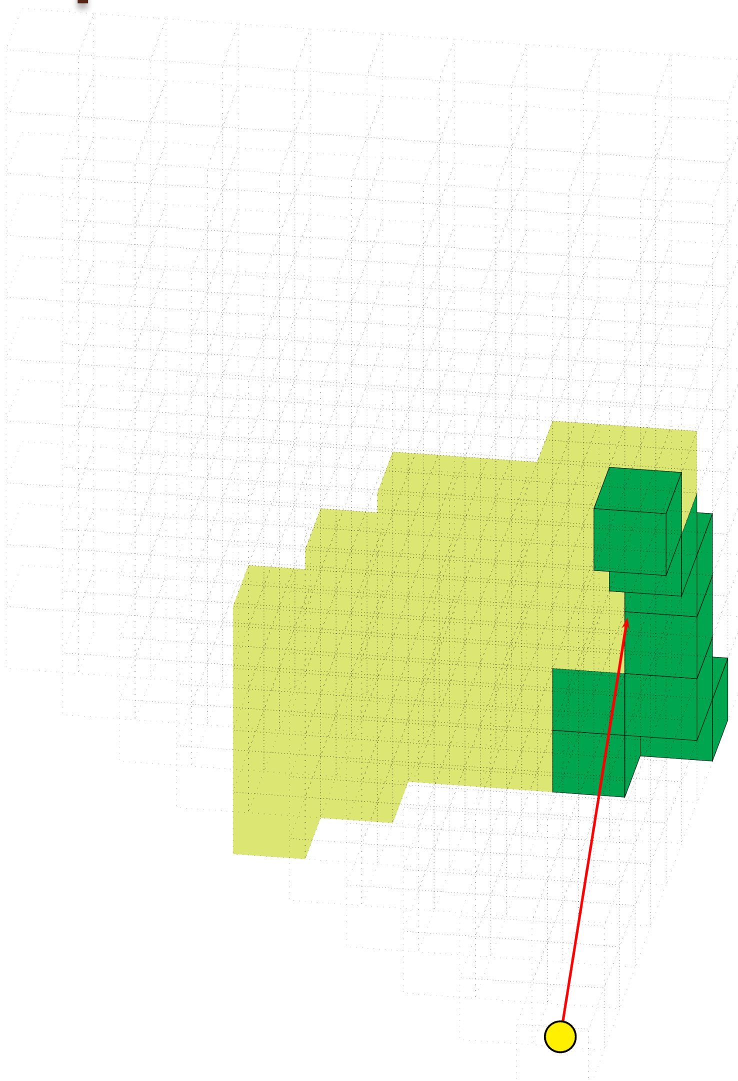
Adaptive time-step scheme



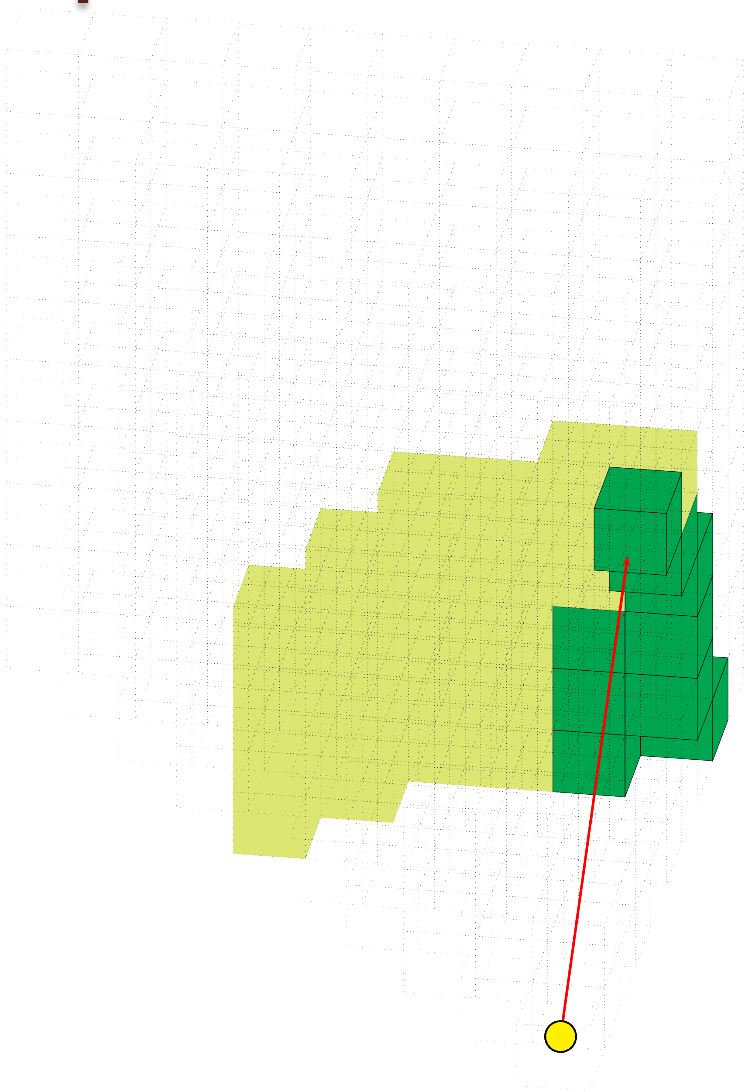
The first slice is complete, we move to the second slice

Adaptive time-step scheme



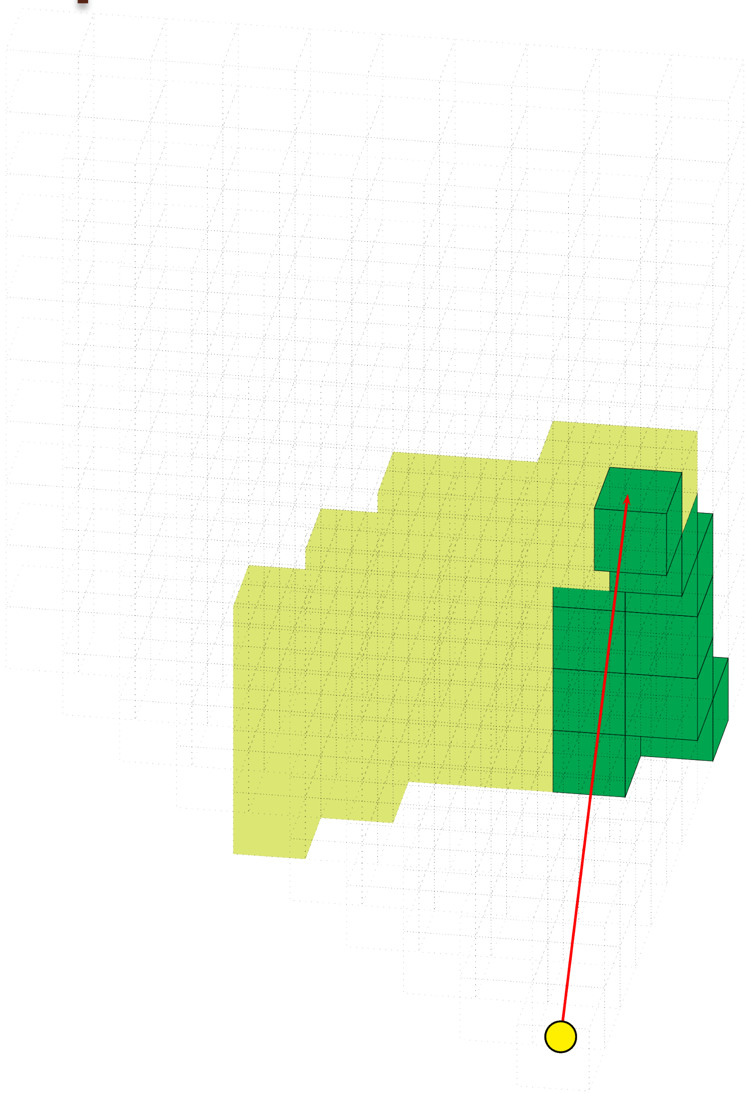
and so on

Adaptive time-step scheme



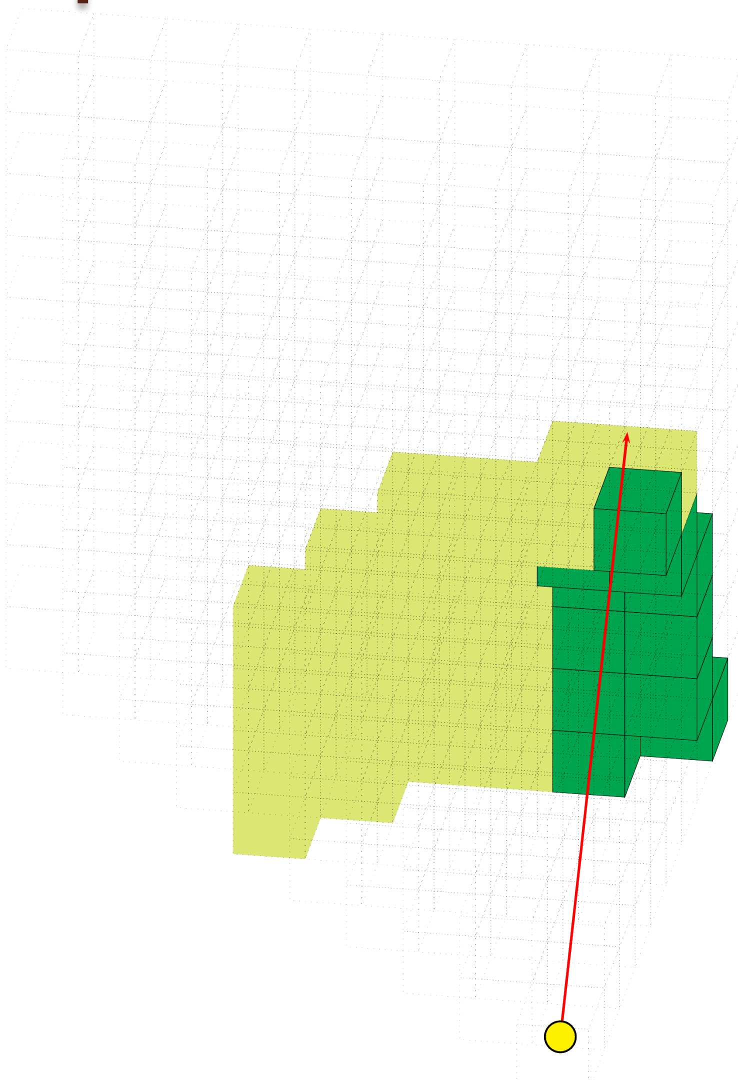
and so on

Adaptive time-step scheme



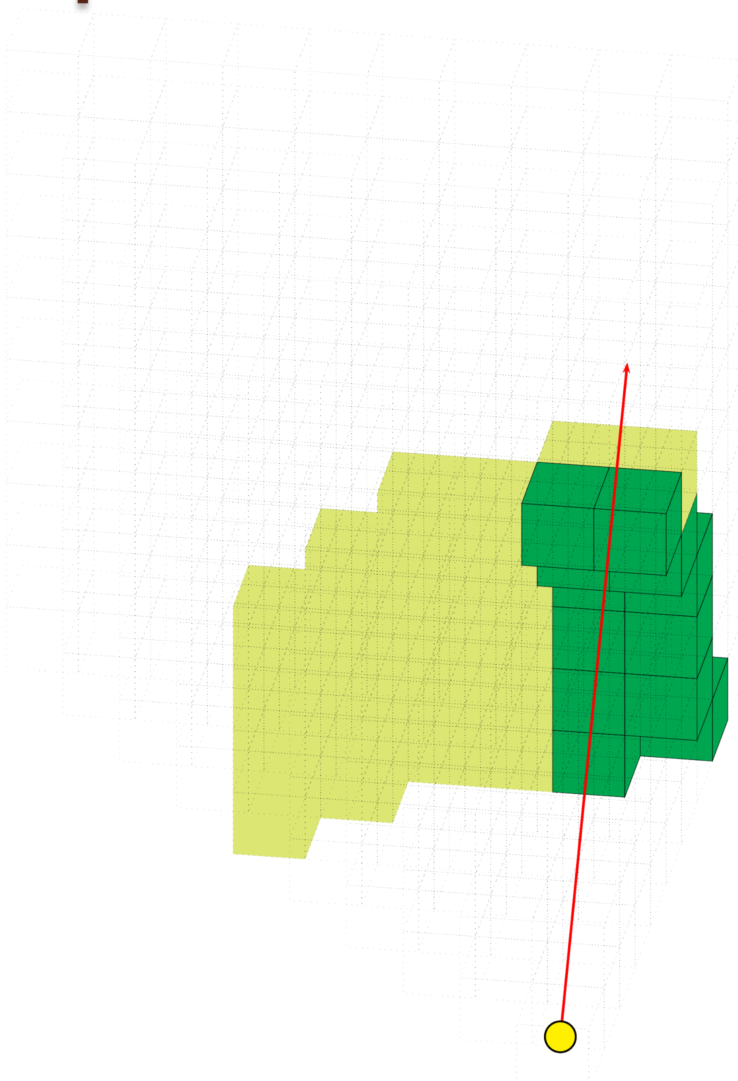
and so on

Adaptive time-step scheme



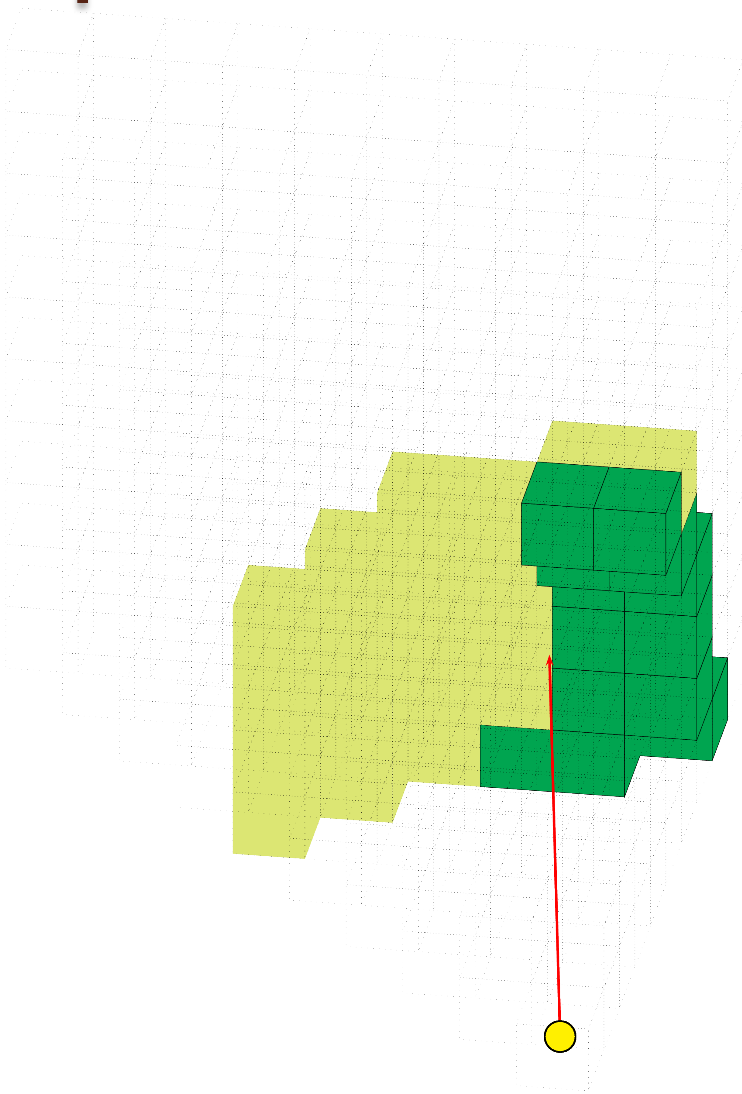
and so on

Adaptive time-step scheme



and so on

Adaptive time-step scheme



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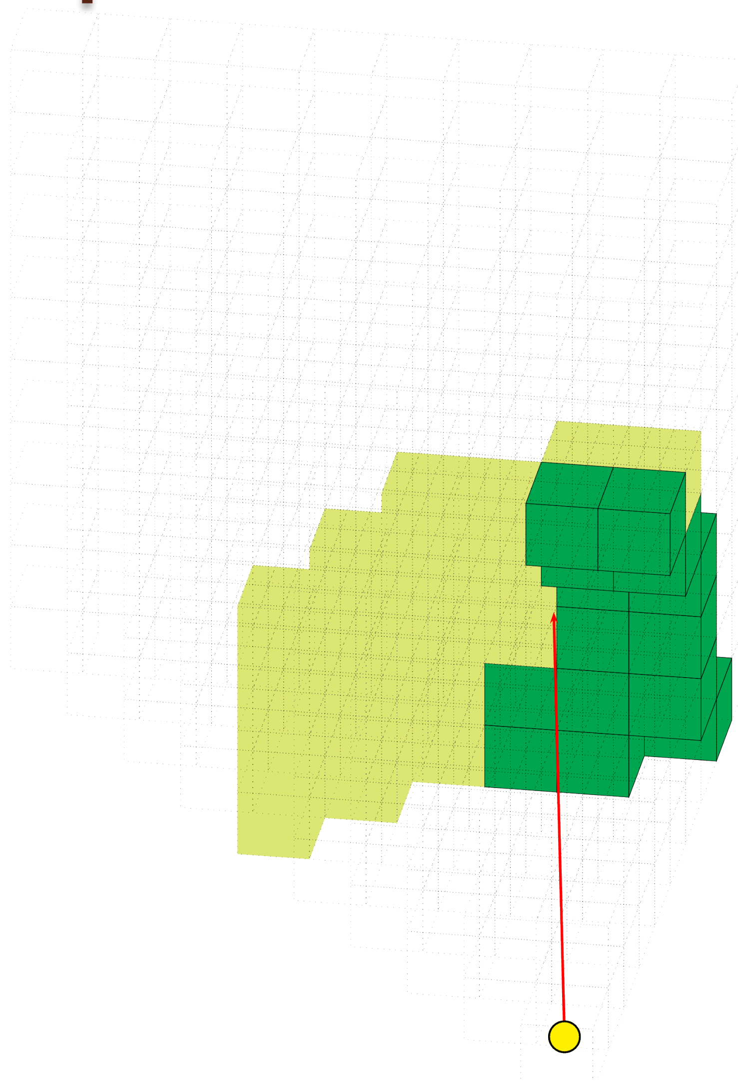
Adaptive time-step scheme



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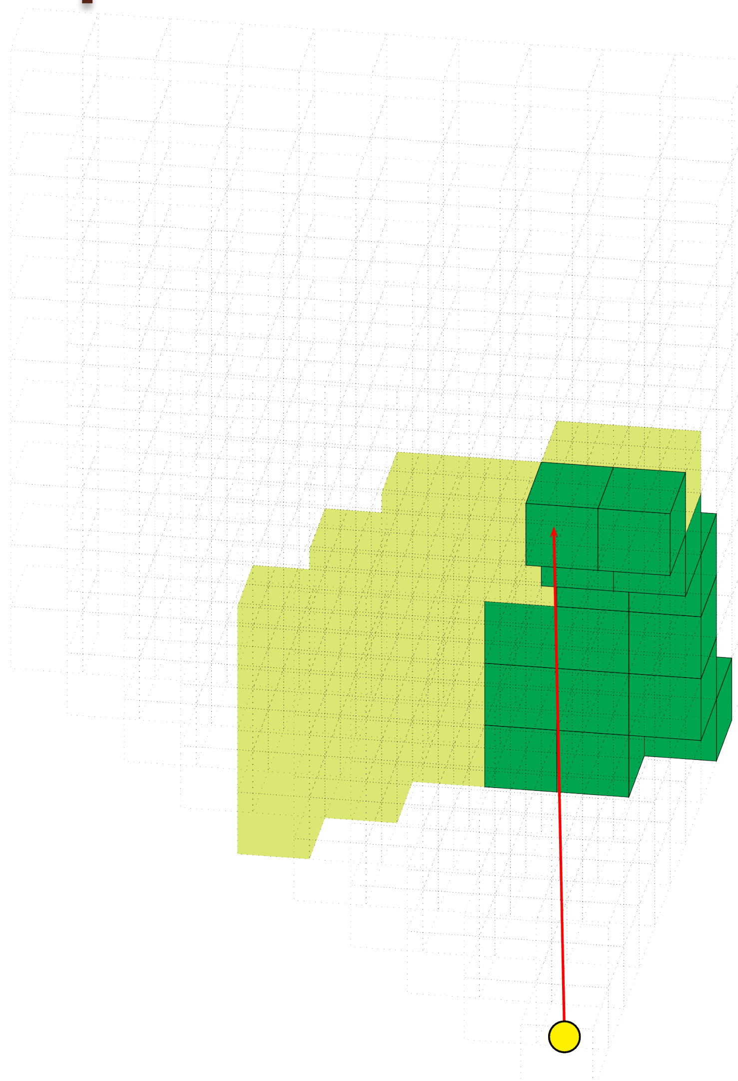
Adaptive time-step scheme



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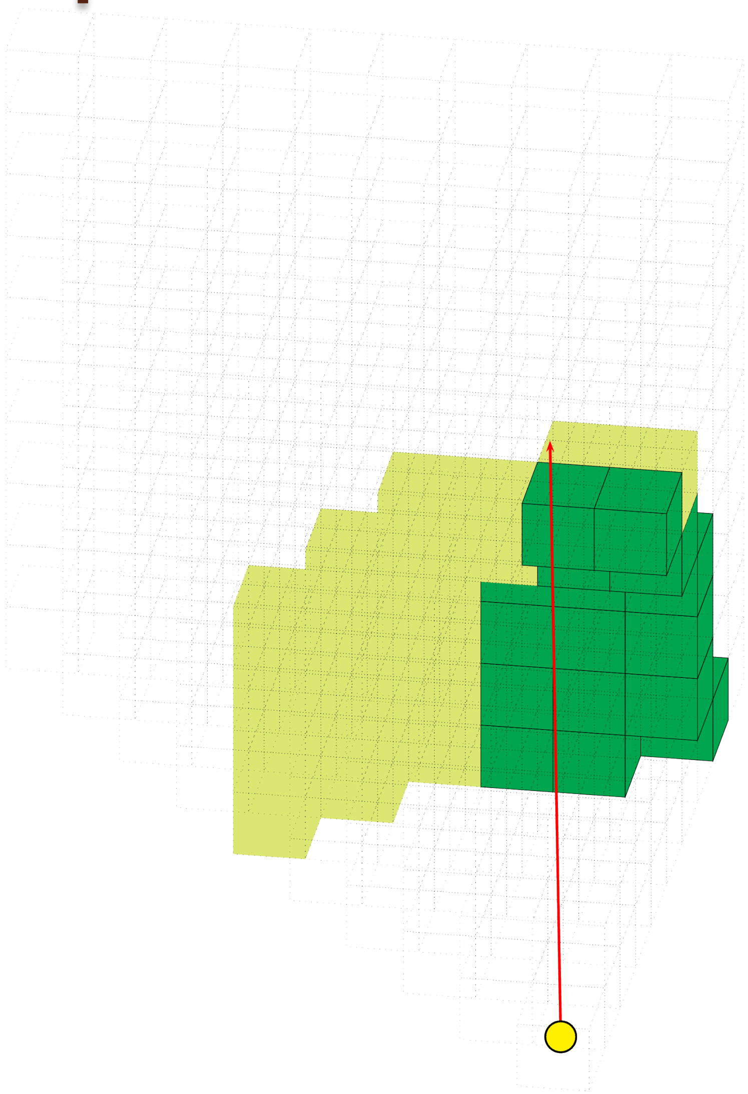
Adaptive time-step scheme



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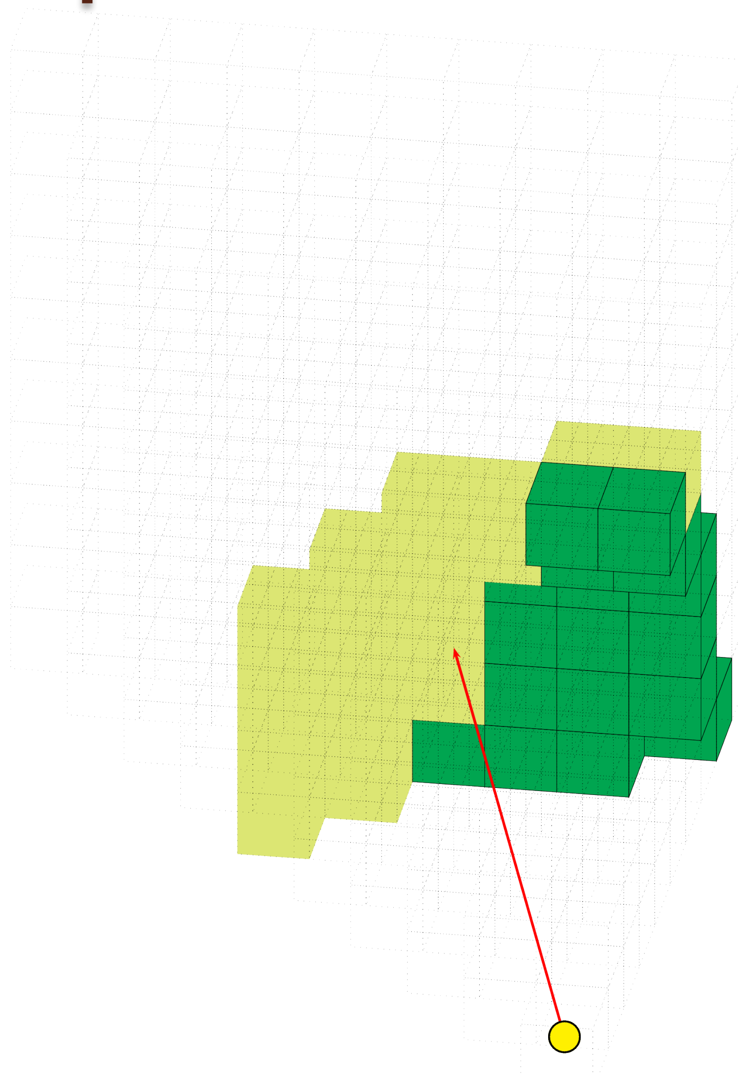
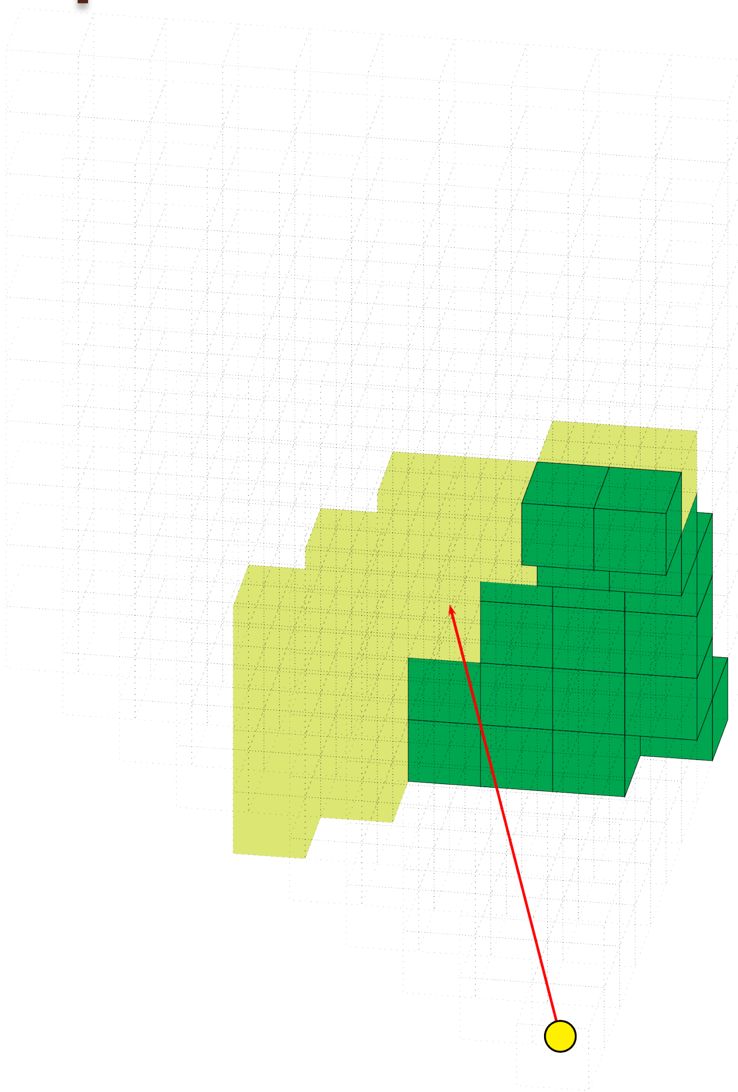
Adaptive time-step scheme



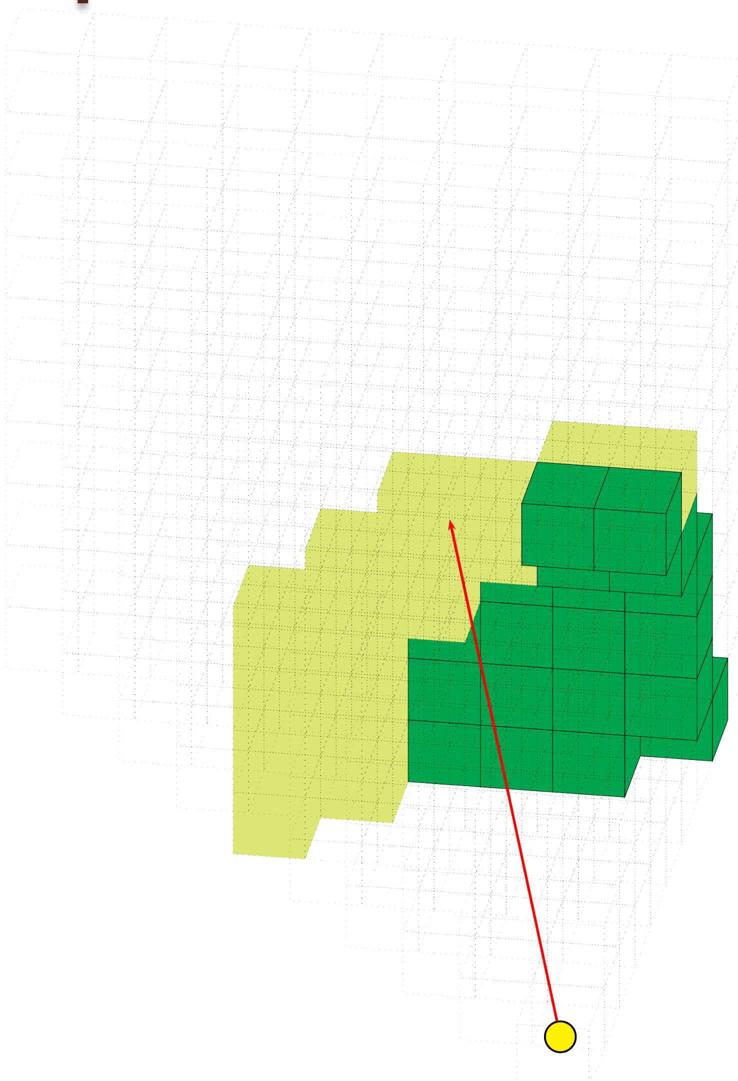
and so on

Adaptive time-step scheme



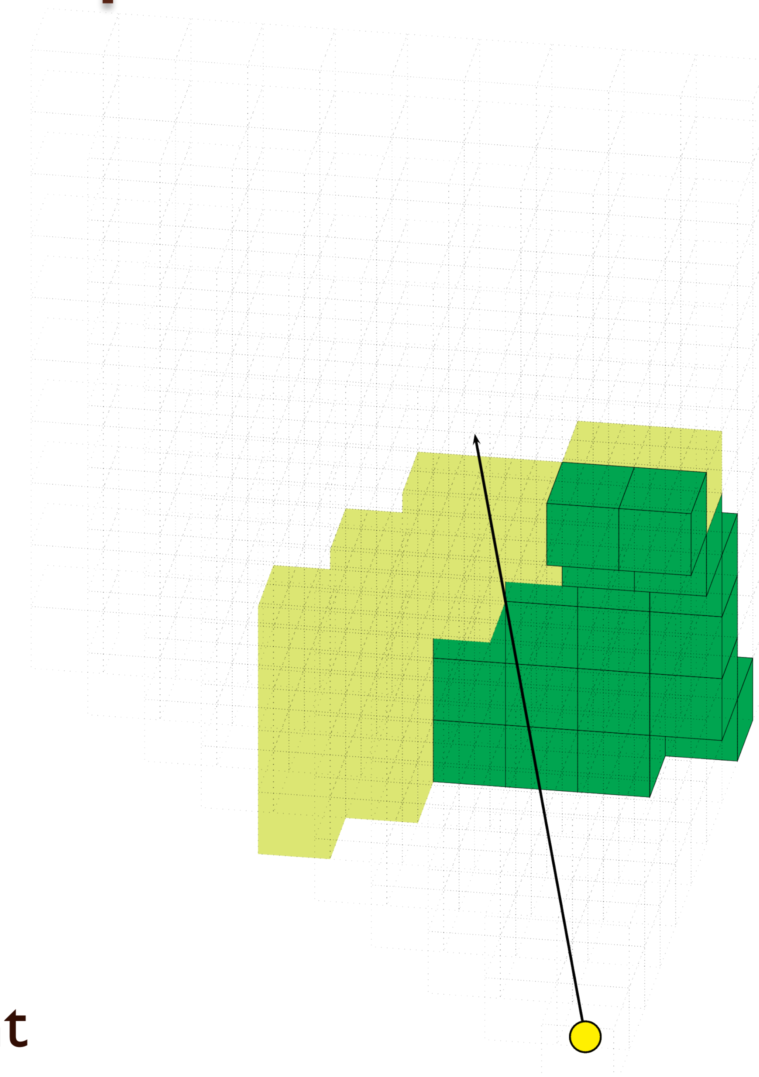
and so on

Adaptive time-step scheme



and so on

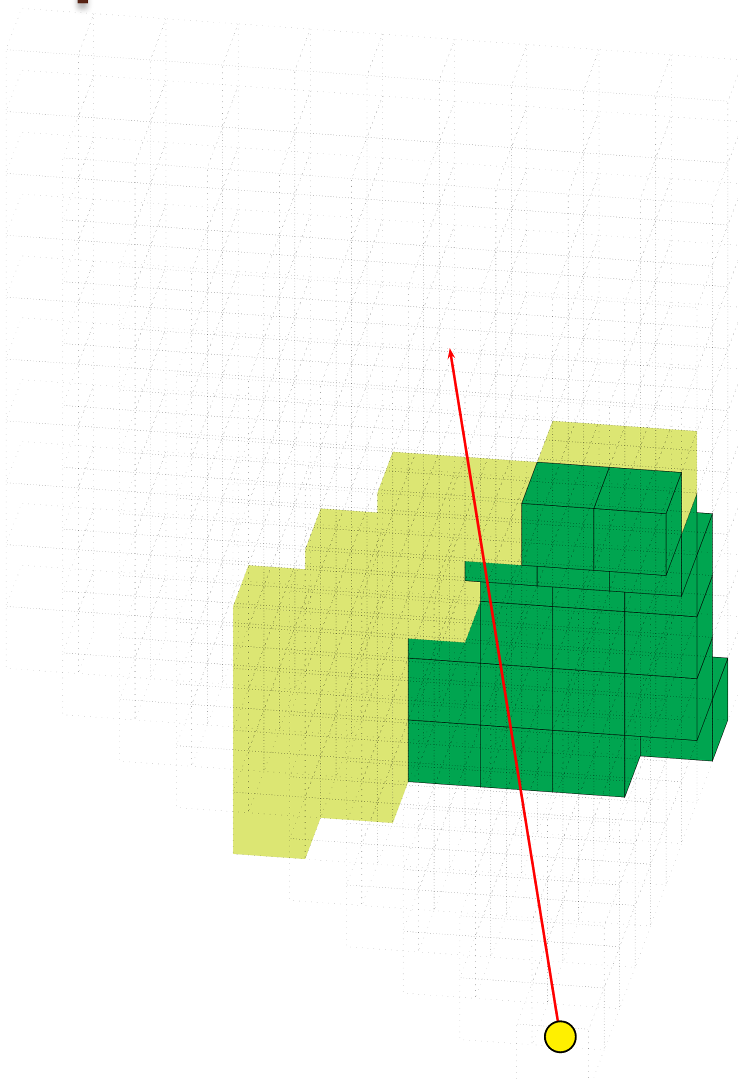
Adaptive time-step scheme



and so on

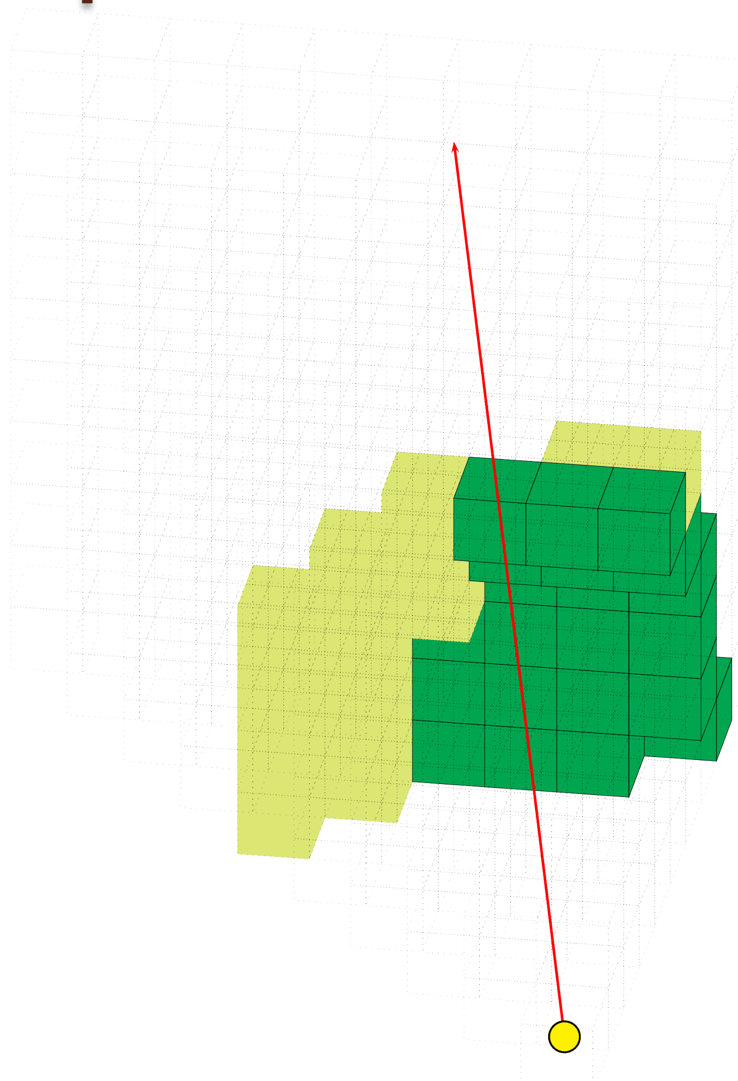
(This ray does not meet ionization front cell)

Adaptive time-step scheme



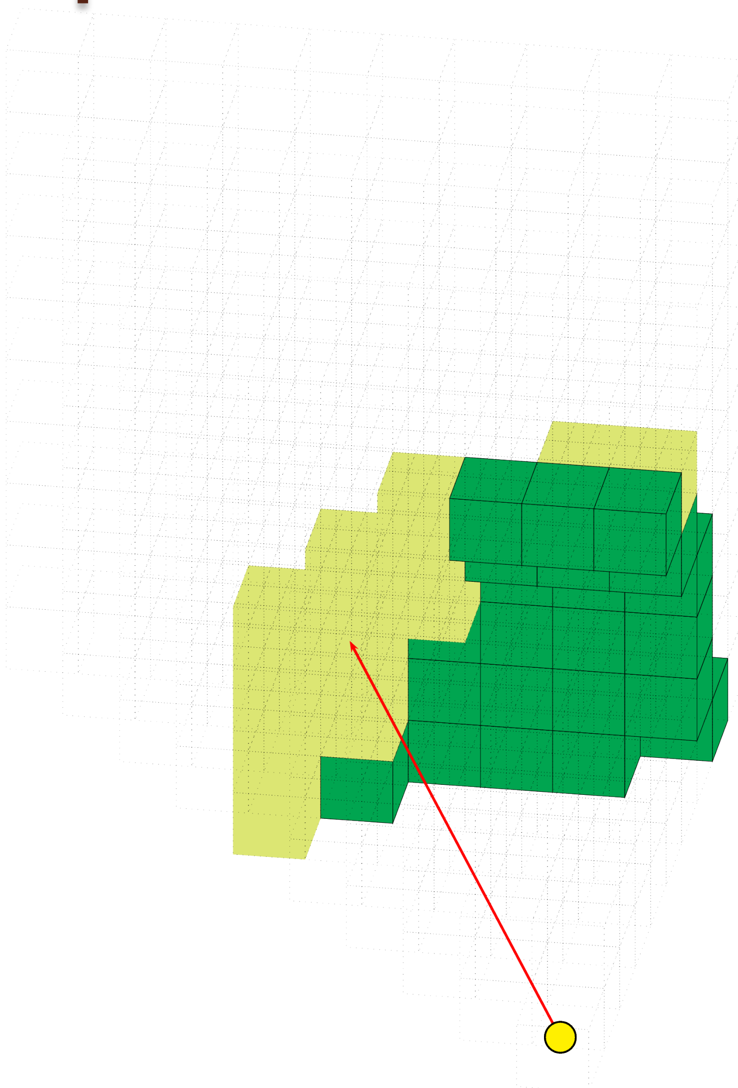
and so on

Adaptive time-step scheme



and so on

Adaptive time-step scheme



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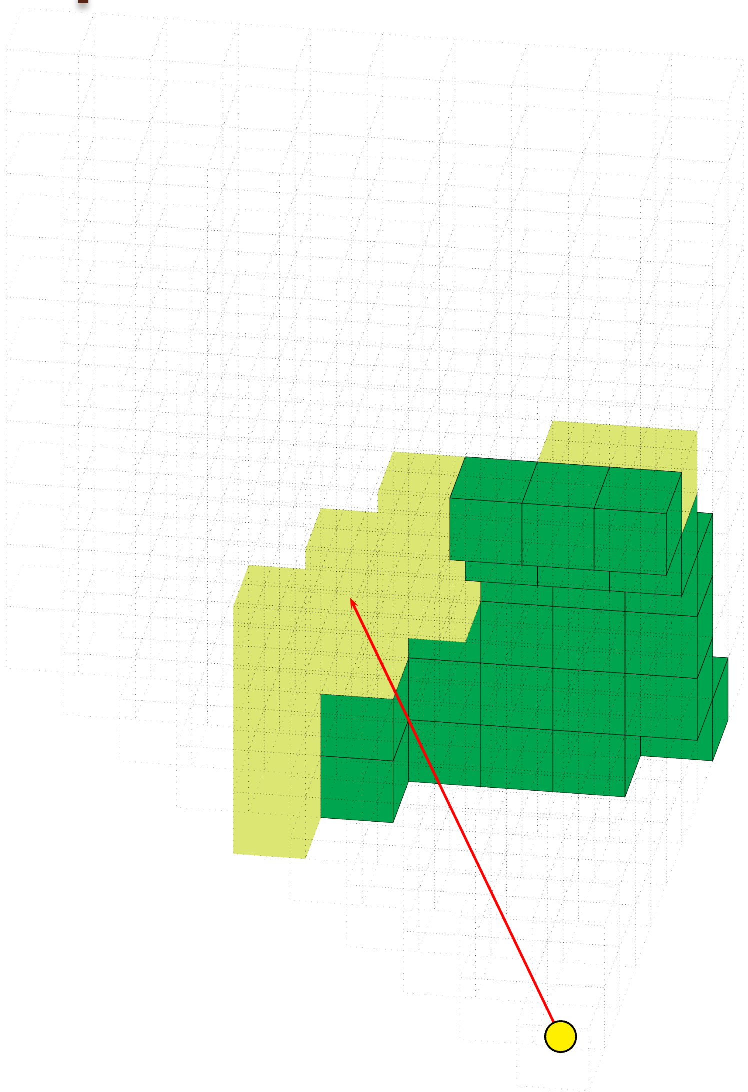
Adaptive time-step scheme



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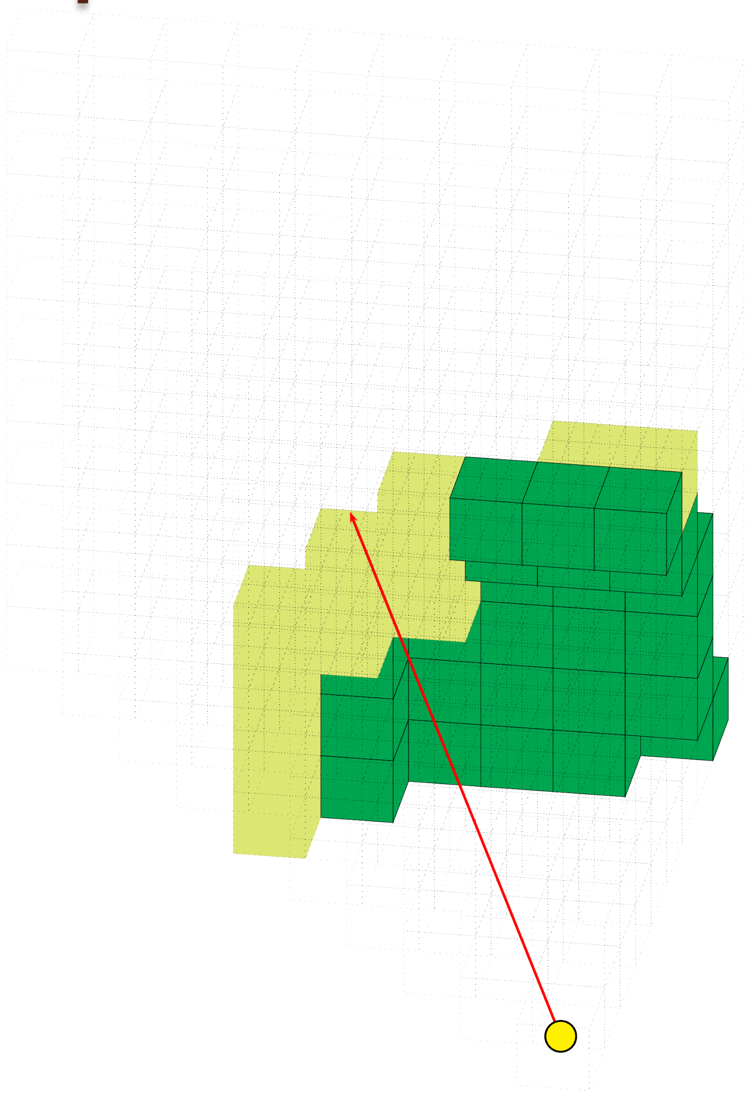
Adaptive time-step scheme



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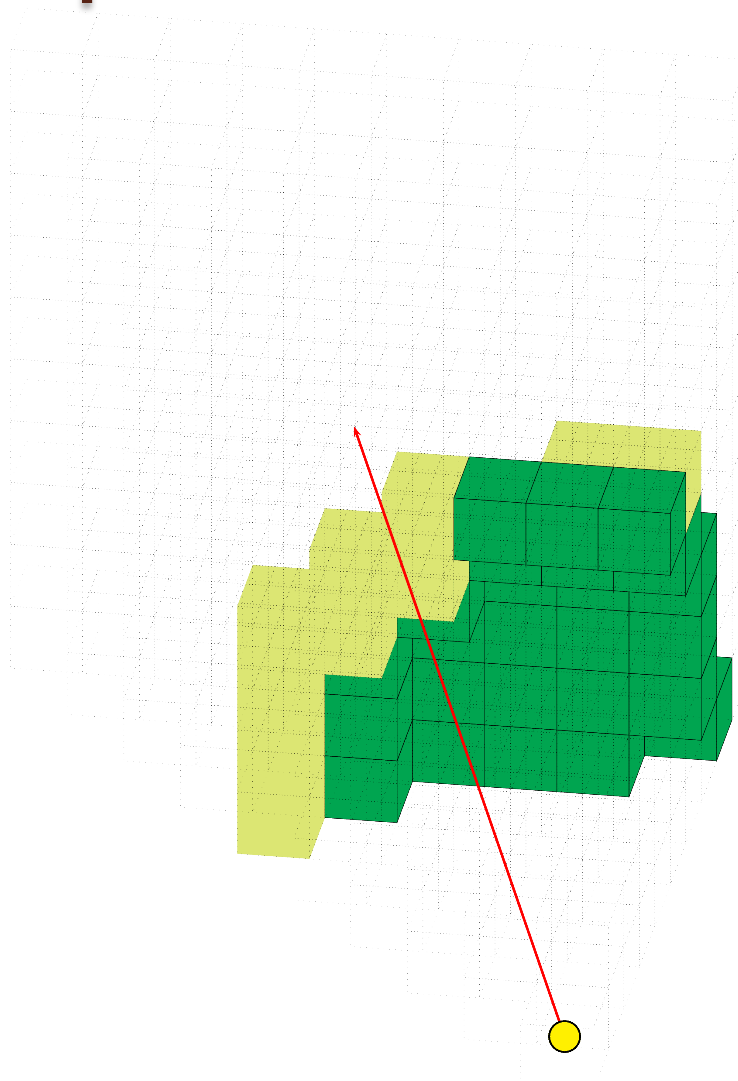
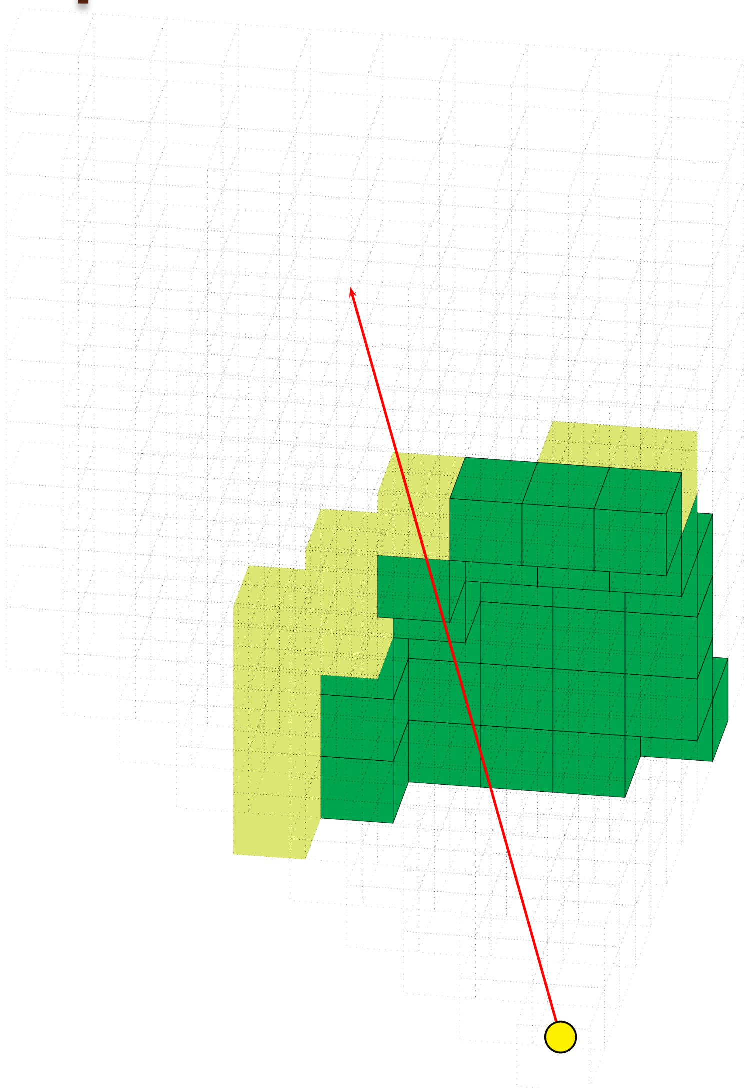
Adaptive time-step scheme



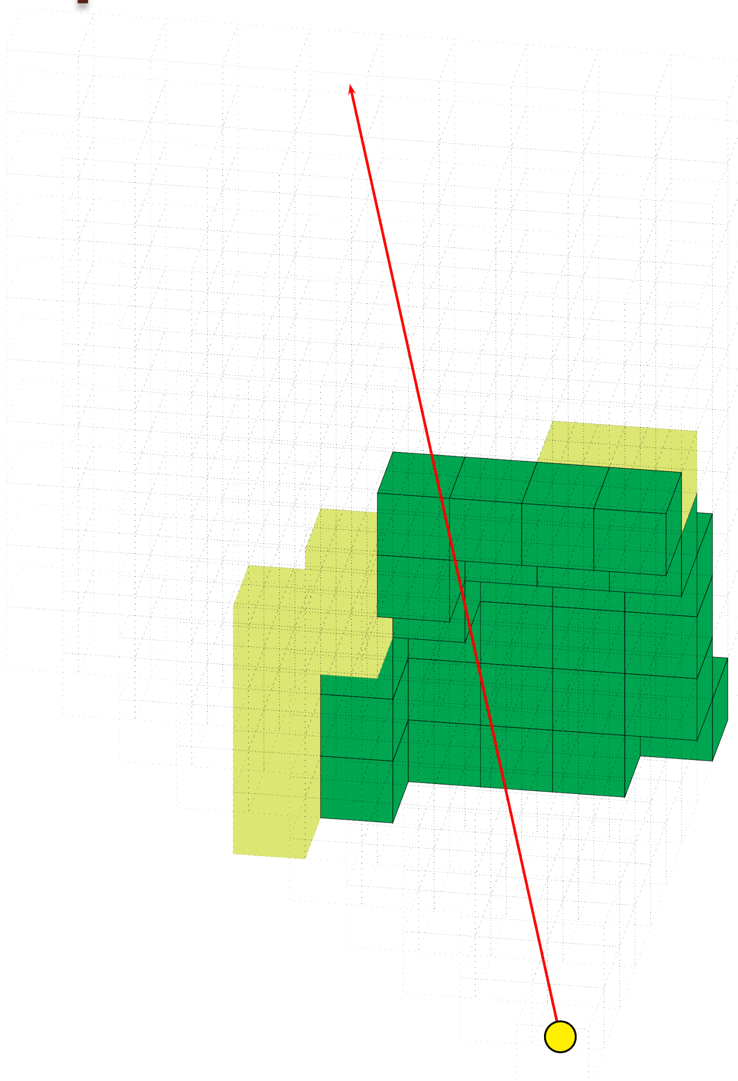
and so on

Adaptive time-step scheme



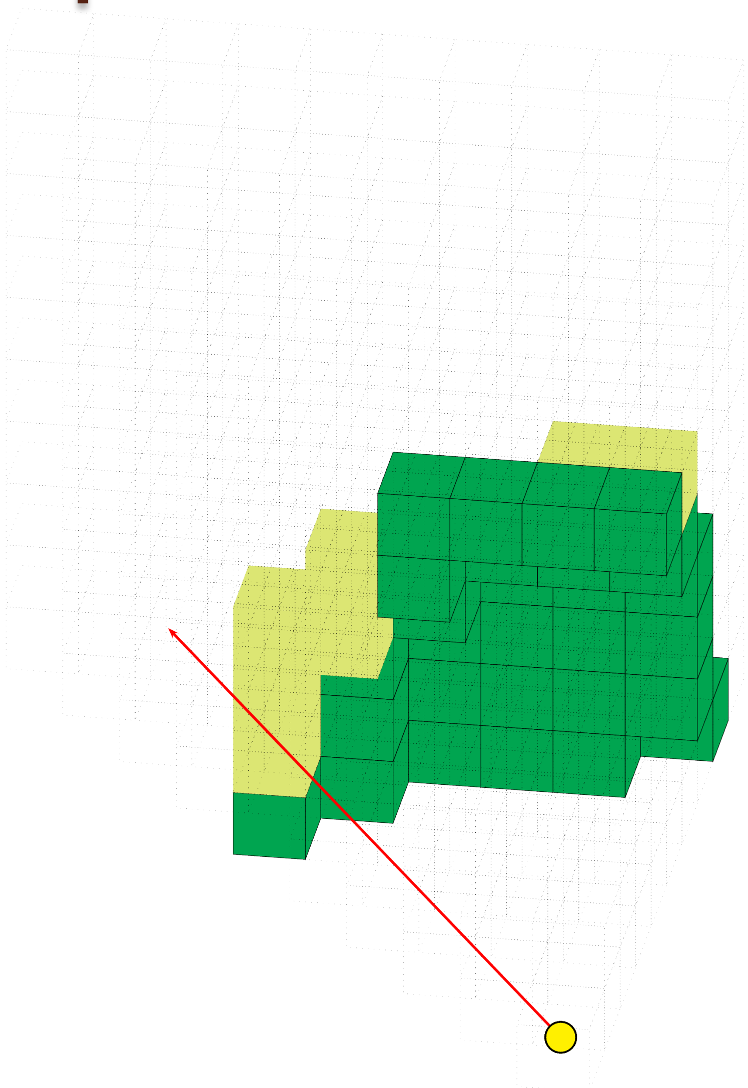
and so on

Adaptive time-step scheme



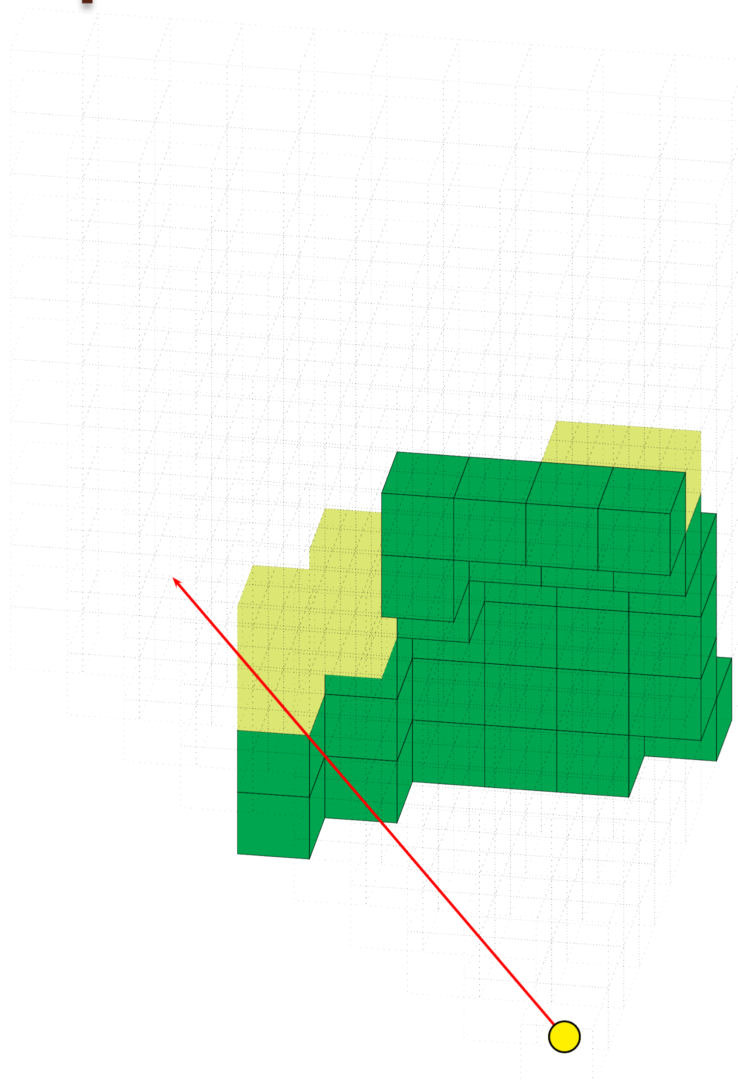
and so on

Adaptive time-step scheme



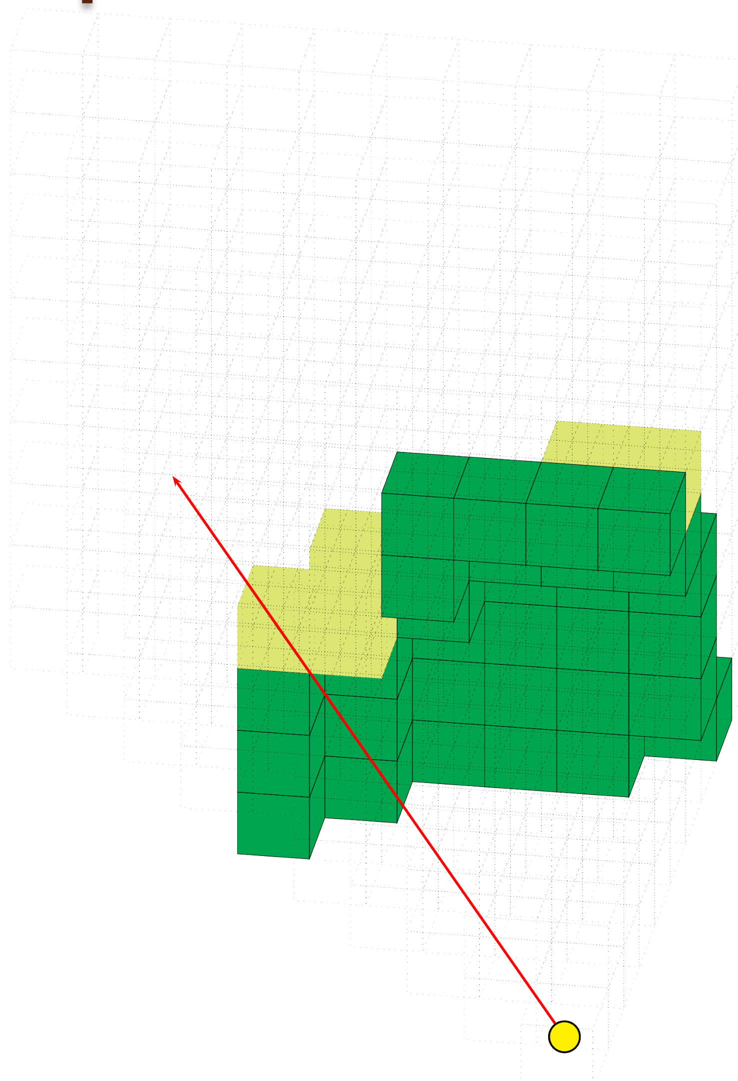
and so on

Adaptive time-step scheme



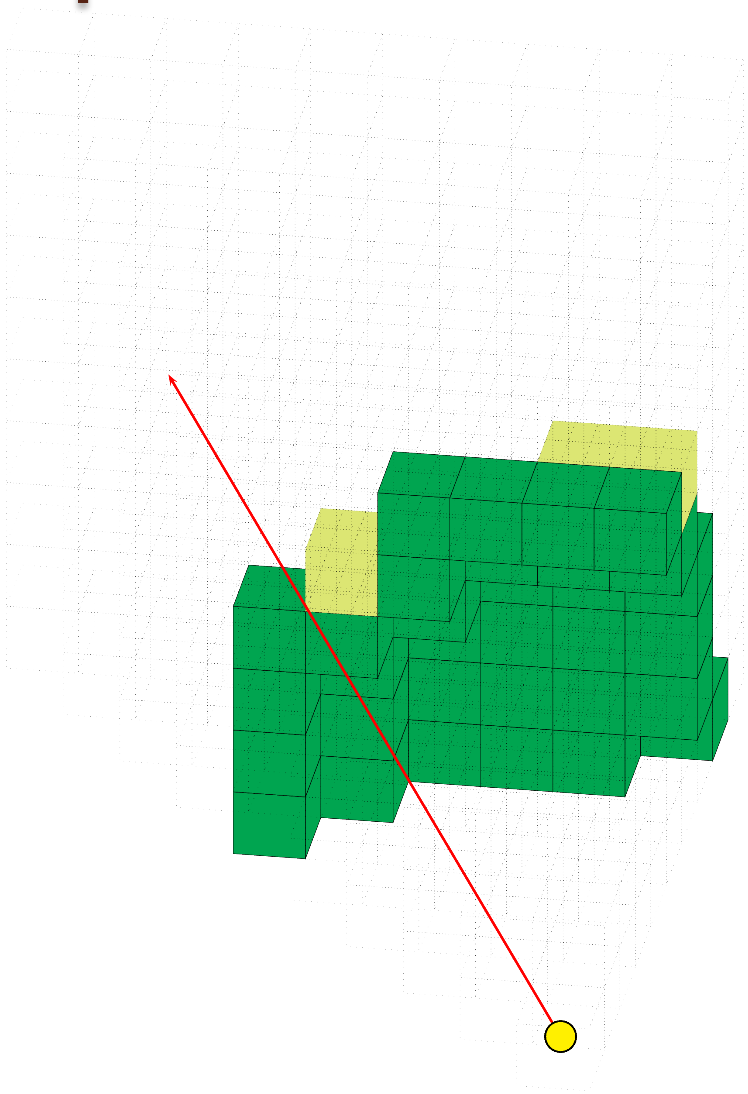
and so on

Adaptive time-step scheme

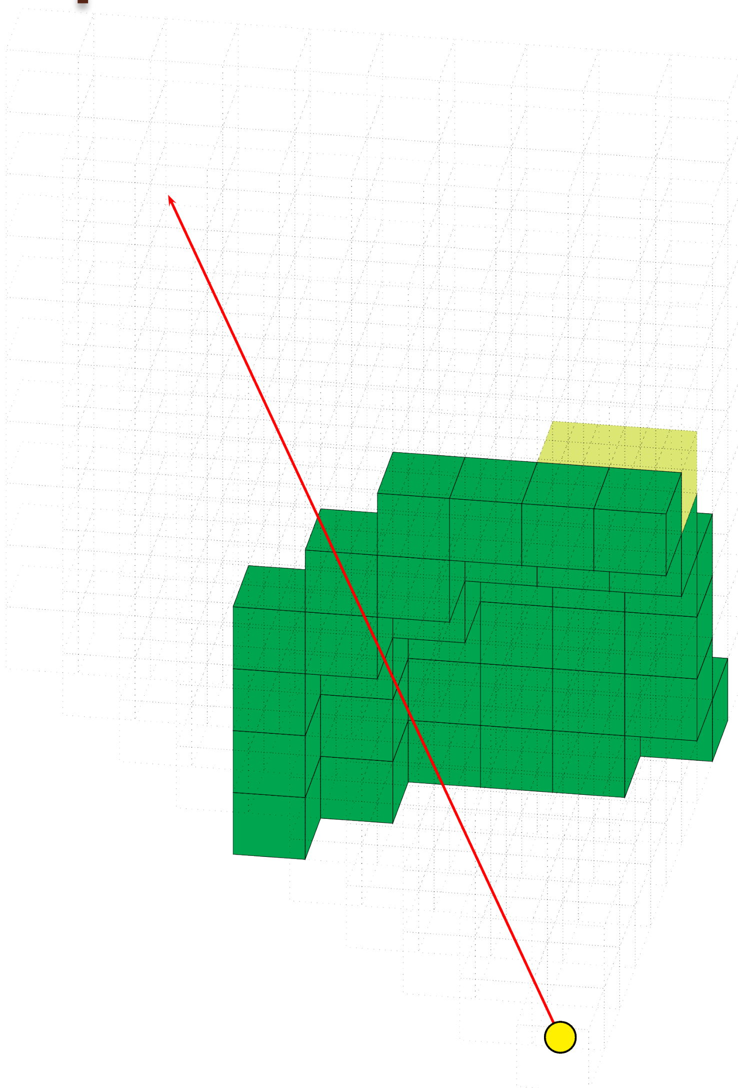


and so on

Adaptive time-step scheme



Adaptive time-step scheme

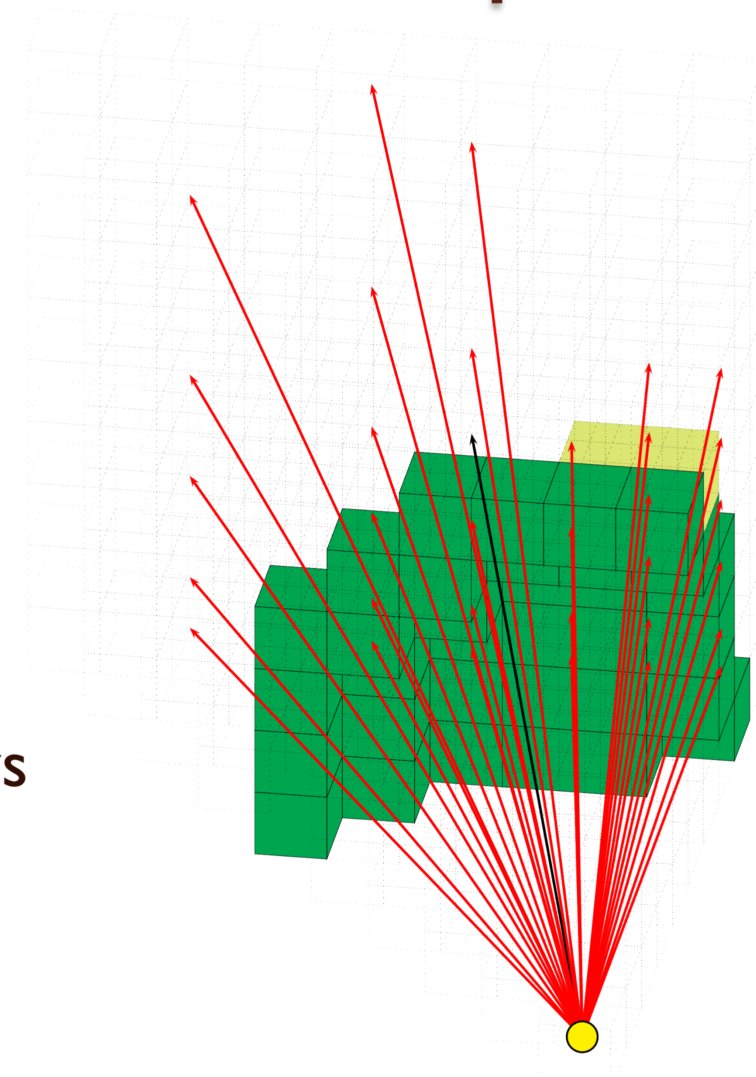


and so on

Trick 1 – Adaptive timestep

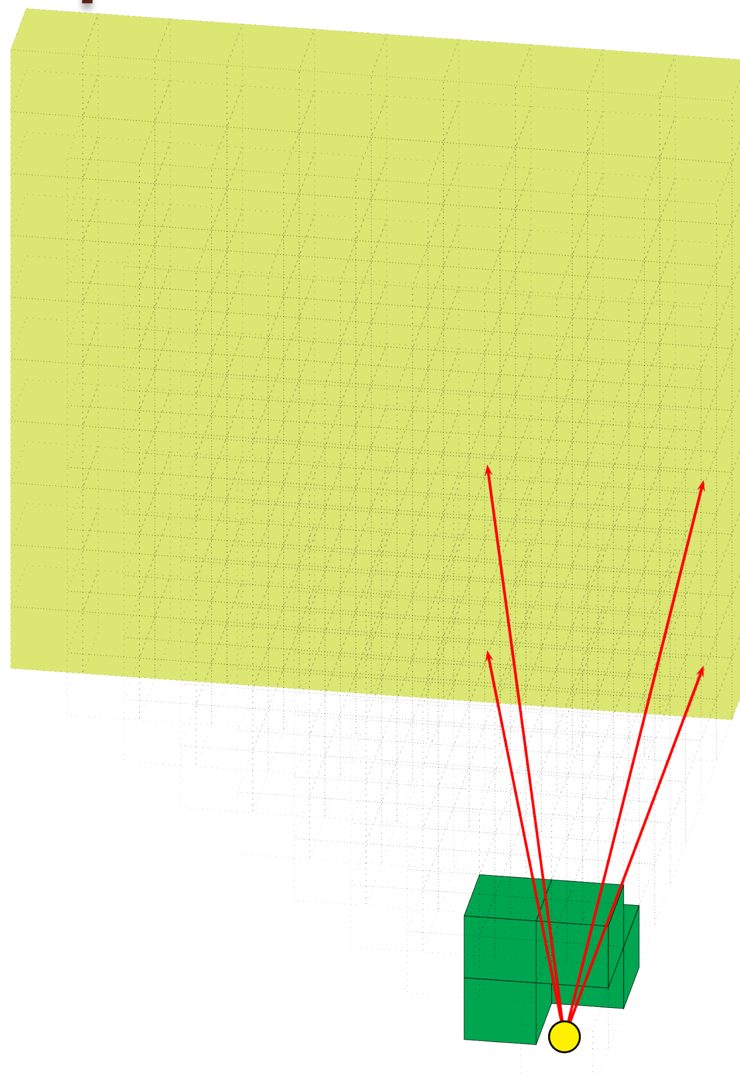
Finish!

These are all the rays that we sent, they are less numerous than the previous one



Adaptive time-step scheme

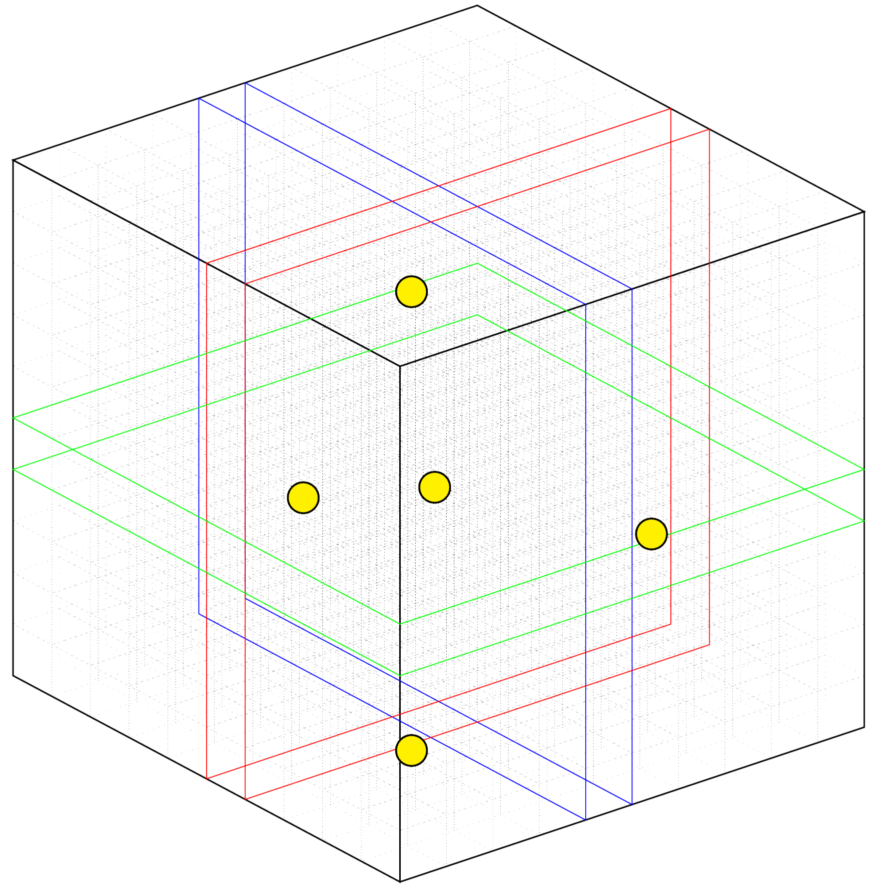
In an extreme case, the source generates a small ionized bubble while other sources generate bigger ionized bubble. However, we use only 4 rays to complete the problem



Asynchronous evolution

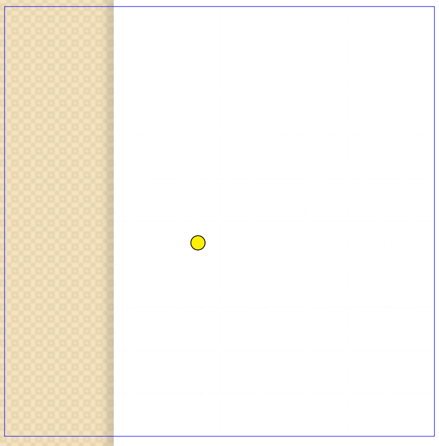
- Given heating timescale << recombination timescale
- Time-step ~ heating timescale is essential to neutral cells
- Time-step ~ recombination timescale is enough for ionized cells
- Applying short time-step to ionized cells is a waste
- Asynchronous evolution : applying long time-step to ionized cells and short time-step to neutral cells
- At the intermediate stage, the clocks of ionized cells and neutral cells are not synchronous, but will finally be synchronized

Asynchronous evolution

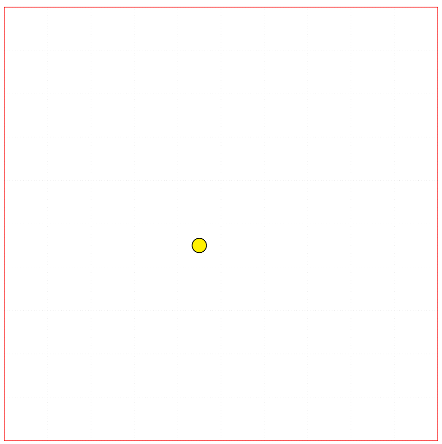


- Given a grid of gas and 5 sources.
- I want to see what happens after Δt
- First, the grid evolves with a short time-step Δt_1

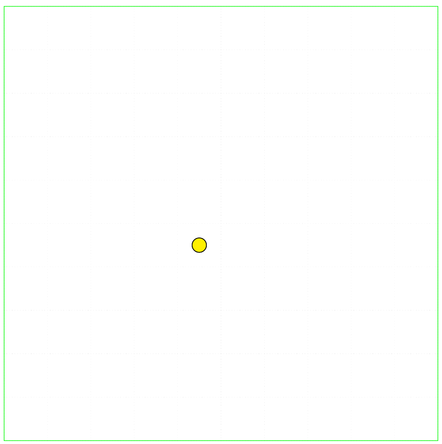
Asynchronous evolution



y-z plane

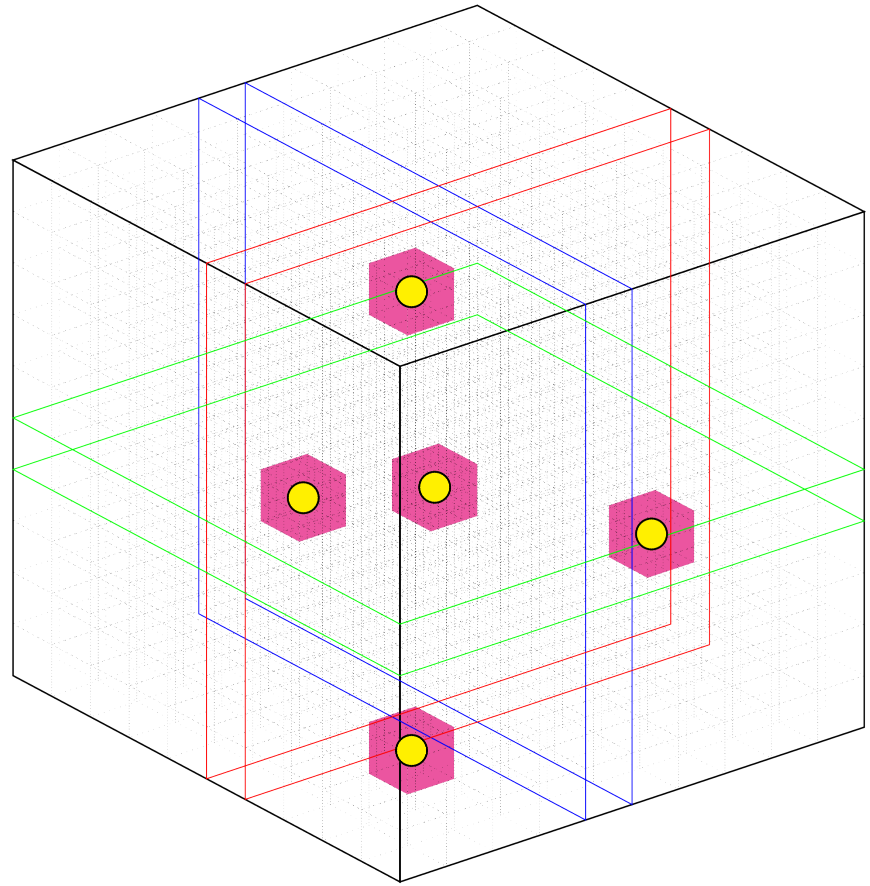


x-z plane



x-y plane

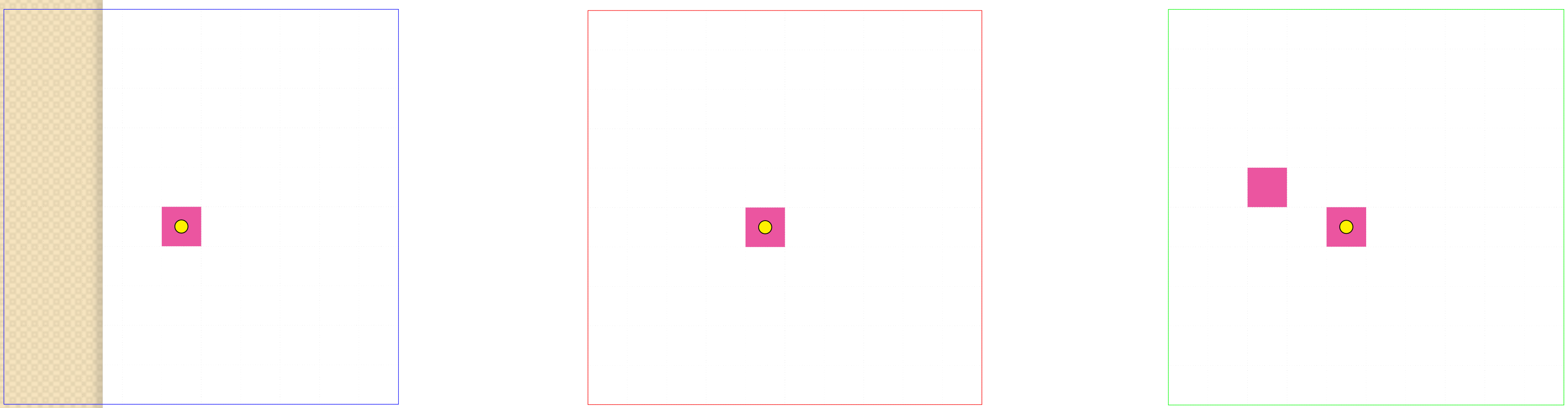
Asynchronous evolution



The pink cells are ionized

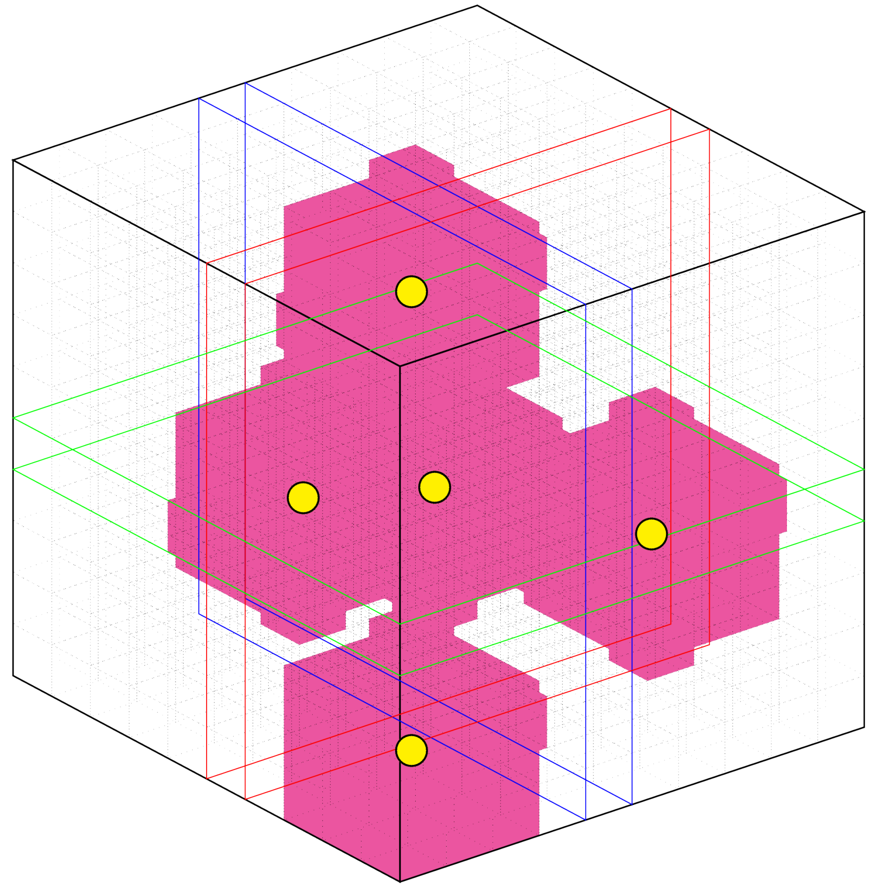
Next, I apply short time-step Δt_2 to the neutral cells and $\Delta t - \Delta t_1$ to the ionized cell

Asynchronous evolution



y-z plane x-z plane x-y plane

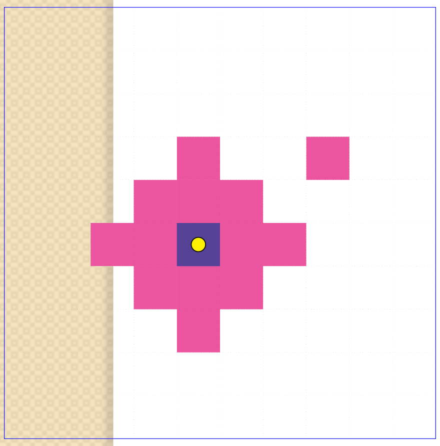
Asynchronous evolution



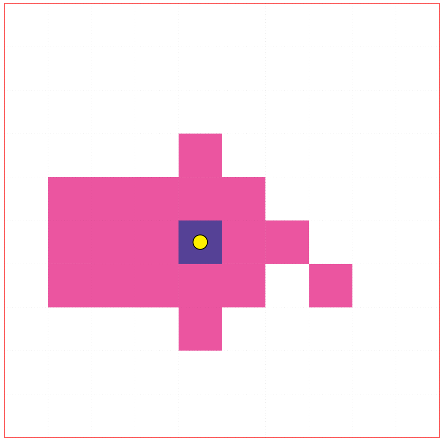
More cells are ionized

Next, I apply short time-step Δt_3 to the neutral cells and $\Delta t - \Delta t_2$ to the newly ionized cell

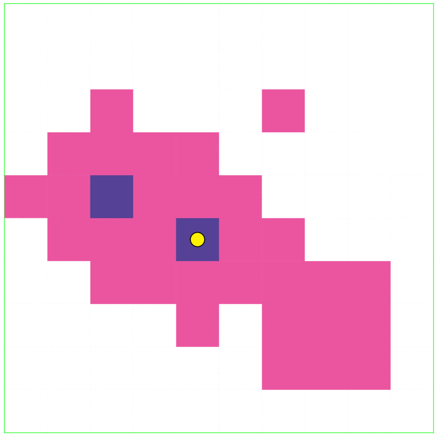
Asynchronous evolution



y-z plane

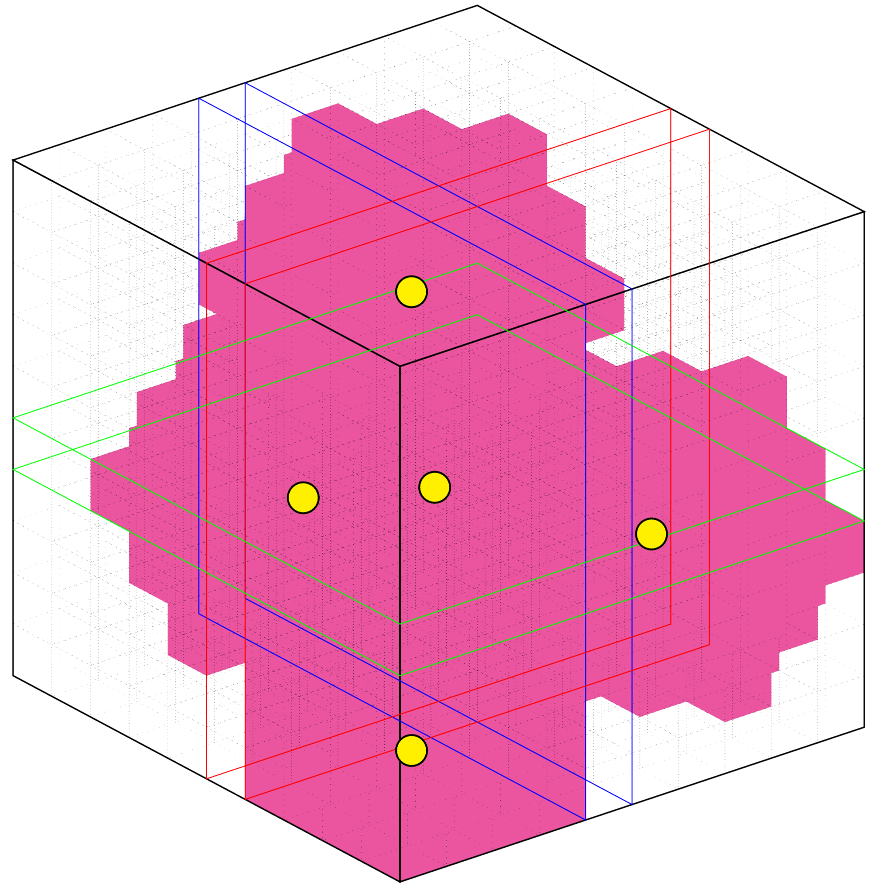


x-z plane



x-y plane

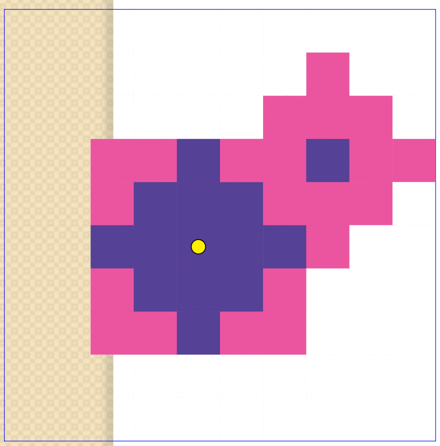
Asynchronous evolution



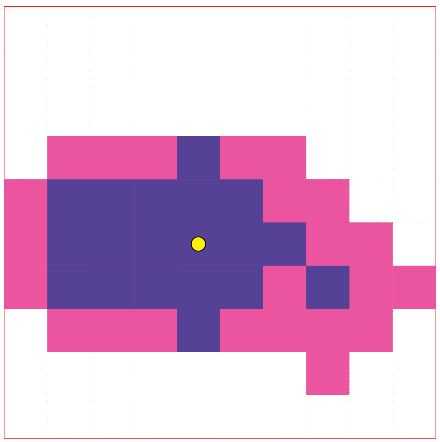
More cells are ionized

Next, I apply short time-step Δt_4 to the neutral cells and $\Delta t - \Delta t_3$ to the newly ionized cell

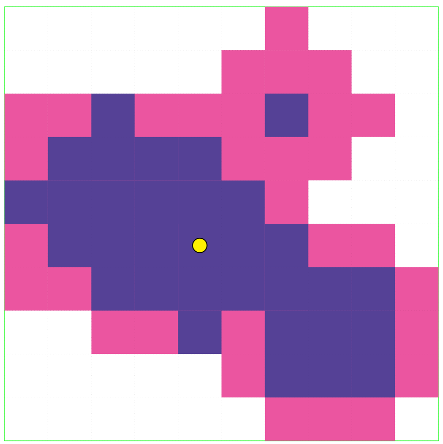
Asynchronous evolution



y-z plane

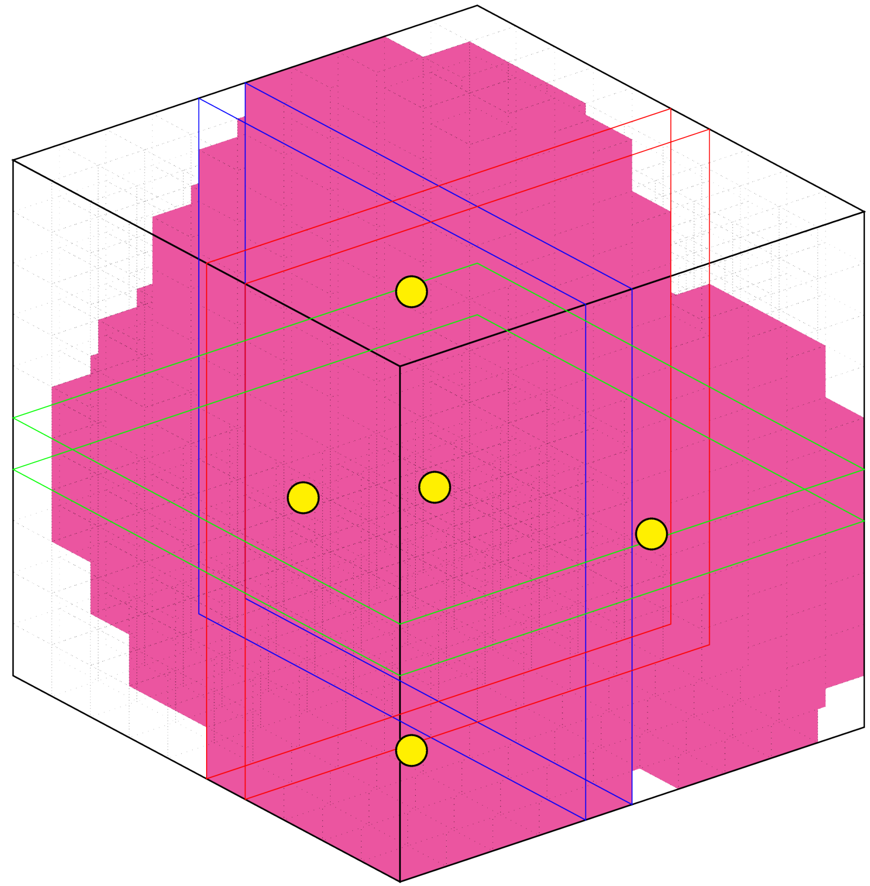


x-z plane



x-y plane

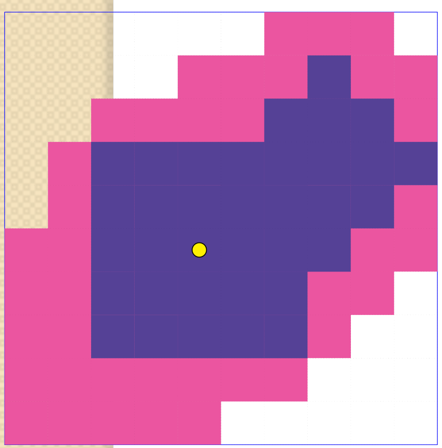
Asynchronous evolution



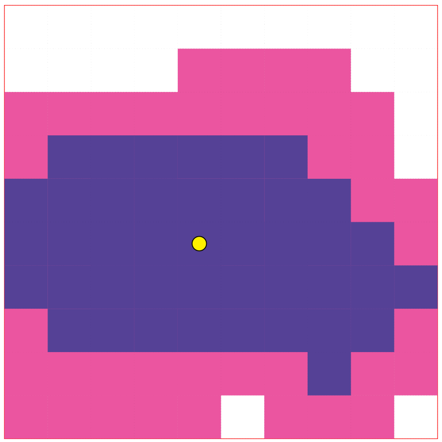
More cells are ionized

Next, I apply short time-step Δt_5 to the neutral cells and $\Delta t - \Delta t_4$ to the newly ionized cell

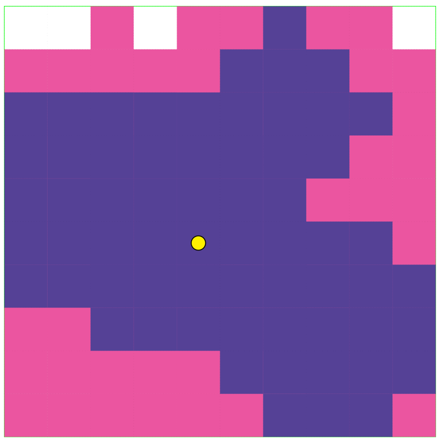
Asynchronous evolution



y-z plane



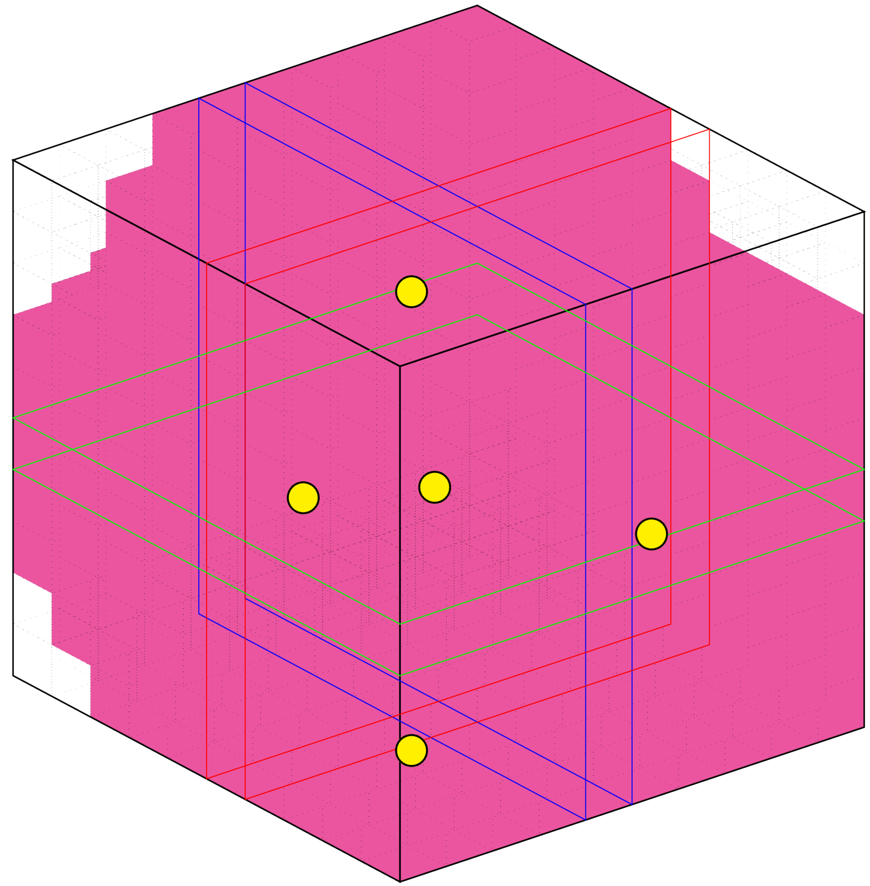
x-z plane



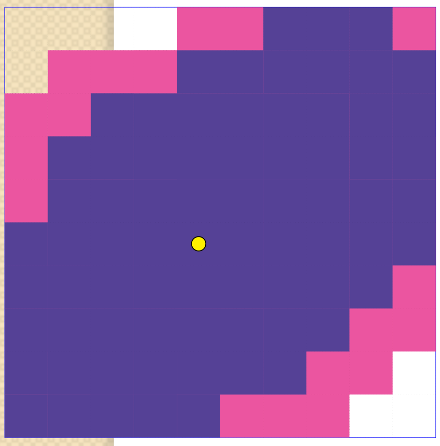
x-y plane

Asynchronous evolution

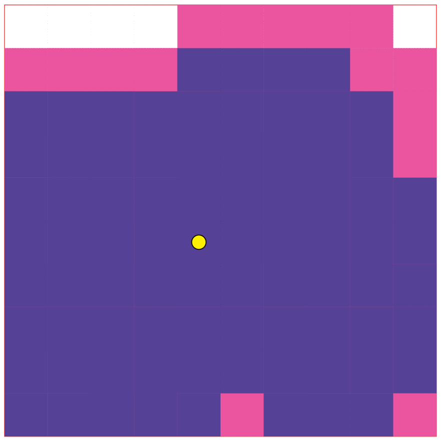
Now Δ t has passed. By allowing cells evolve with different time-step, the actual number of evolutions is reduced. Computational cost is also reduced!



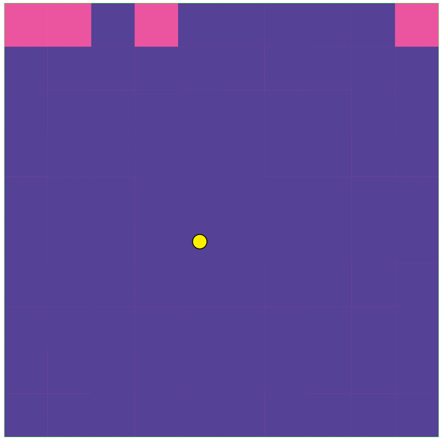
Asynchronous evolution



y-z plane



x-z plane



x-y plane

Summary

To accurately calculate thermal evolution, short time-step is required.

Finding correct time-step can be expensive

Solved by strategy - Adaptive time-step

Evolution of entire domain with short time-step is very expensive

Solved by strategy - Asynchronous evolution